REACTION FORMULATION FOR RADIATION AND SCATTERING
FROM PLATES, CORNER REFLECTORS AND
DIELECTRIC-COATED CYLINDERS

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ABSTRACT

The reaction concept is employed to formulate an integral equation for radiation and scattering from plates, corner reflectors, and dielectric-coated conducting cylinders. The surface-current density on the conducting surface is expanded with subsectional bases. The dielectric layer is modeled with polarization currents radiating in free space. Maxwell's equation and the boundary conditions are employed to express the polarization-current distribution in terms of the surface-current density on the conducting surface. By enforcing reaction tests with an array of electric test sources, the moment method is employed to reduce the integral equation to a matrix equation. Inversion of the matrix equation yields the current distribution, and the scattered field is then obtained by integrating the current distribution.

This report presents the theory, computer program and numerical results for radiation and scattering from plates, corner reflectors, and dielectric-coated conducting cylinders.

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CHAPTER I
INTRODUCTION

Low-frequency solutions for radiation and scattering from cylinders have been reported in several published papers [1-8]. Among them, Mei and Van Bladel [1] employed a point-matching procedure to solve the electric-field integral equation and the magnetic-field integral equation for transverse-magnetic and transverse-electric incident waves, respectively. In the point-matching procedure the surface current distribution was expanded into rectangular-pulse bases and the appropriate boundary conditions were enforced at discrete points on the conducting surface of the cylinder. Richmond [4] has developed a wire-grid array model for cylinders with transverse-magnetic incident wave and shown that if a sufficiently great number of wires is employed, the scattering pattern approaches that of a solid cylinder of the same contour. Richmond [9] has also developed a piecewise-sinusoidal reaction formulation for electromagnetic radiation and scattering problems involving cylinders with non-circular cross section for the transverse-electric incident wave. No solution, however, has been published for non-circular cylinders with a dielectric coating.

For three-dimensional problems, two methods are available for electromagnetic modeling of a continuous conducting surface with arbitrary shape: the wire-grid model [10] and the surface-current model [11,12] with rectangular-pulse bases. Both methods have similar limitations with the maximum cell width restricted to approximately one-tenth of a wavelength. Unless the conducting body is symmetric or is a figure of revolution, computer storage requirements have limited the conducting surface area to one or two square wavelengths.

In this dissertation, Rumsey's [13] reaction concept is employed to formulate an integral equation for scattering by plates, corner reflectors and dielectric-coated cylinders with noncircular cross section. In the reaction formulation, the surface-current density on the conducting surface is expanded with suitable bases. The dielectric layer is modeled with the equivalent polarization currents radiating in free space. Maxwell's equations and the boundary conditions are employed to express the polarization-current distribution in terms of the surface-current density on the conducting surface. By enforcing reaction tests with an array of electric test sources, the moment method is employed to reduce the integral equation to a matrix equation. Numerical solution of this system yields a stationary result for the samples of the current distribution. Finally, the quantities of interest such as the gain, far-field pattern, and the radar cross section are determined from the current distribution. With perfect conductivity, the analysis presented in this dissertation is valid for open as well as closed cylinders. With finite conductivity or with a thin dielectric coating, however, the analysis is restricted to closed cylinders.

The remaining text presents the general theory of the reaction formulation for radiation and scattering from conducting bodies. The time dependence $e^{j\omega t}$ is understood and suppressed. Chapter II presents the detailed theoretical outline of the reaction concept which forms the foundation of this dissertation. Relevant electric sources employed as the expansion functions and the test sources are discussed in Chapter III. Evaluation of the mutual impedances and excitation voltages are considered in Chapter IV and Chapter V. The electromagnetic modeling of the conducting body is an essential step in the reaction formulation. This is described in Chapter VI. The field scattered by a conducting body is the sum of the contributions from all the current modes which appear in the expansion for the surface-current distribution. In Chapter VII the far-field contributions of these sources are discussed. Numerical results for the radar cross section of rectangular plates, corner reflectors and dielectric-coated cylinders are presented in Chapter VIII.

CHAPTER II
THE REACTION INTEGRAL FORMULATION

Two well known integral equations, the electric-field integral equation and the magnetic-field integral equation are usually employed to solve electromagnetic radiation and scattering problems. In this chapter, however, we develop the more general reaction integral equation of Rumsey [13]. It has been noted [14] that the reaction integral equation is more general in the sense that it can be reduced to either the electric-field integral equation or the magnetic-field integral equation if one enforces the reaction integral equation with a set of delta-function electric or magnetic test sources. In the following sections the reaction concept and its application in electromagnetic problems will be examined.

Consider the exterior scattering problem illustrated in Fig. 1a. In the presence of a conducting body, the impressed electric and magnetic currents $(\underline{J}_i, \underline{M}_i)$ generate the electric and magnetic field intensities $(\underline{E}, \underline{H})$. For simplicity, let the exterior medium be free space.

From the surface-equivalence theorem of Schelkunoff [15], the interior field will vanish (without disturbing the exterior field) if we introduce the following surface-current densities

(1) $\underline{J}_s = \hat{n} \times \underline{H}$

(2) $\underline{M}_s = \underline{E} \times \hat{n}$

on the closed surface S of the scatterer. (The unit vector $\hat{n}$ is directed outward on S.) In this situation, illustrated in Fig. 1b, we may replace the scatterer with free space without disturbing the field anywhere.

By definition, the incident field $(\underline{E}_i, \underline{H}_i)$ is generated by $(\underline{J}_i, \underline{M}_i)$ in free space, and the scattered field is:

(3) $\underline{E}_s = \underline{E} - \underline{E}_i$

(4) $\underline{H}_s = \underline{H} - \underline{H}_i$.

When the surface current $(\underline{J}_s, \underline{M}_s)$ radiates in free space, it generates the field $(\underline{E}_s, \underline{H}_s)$ in the exterior and $(-\underline{E}_i, -\underline{H}_i)$ in the interior region. This result, illustrated in Fig. 1c, is deduced from Fig. 1b and the superposition theorem.

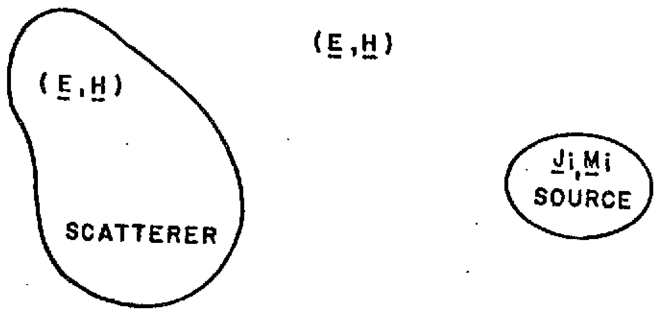

Fig. 1a--The source ($\underline{J}_i$, $\underline{M}_i$) generates the field ($\underline{E}$,$\underline{H}$) with scatterer.

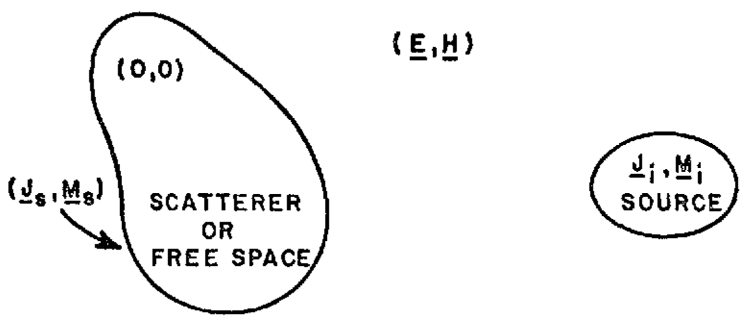

Fig. 1b--The interior field vanishes when the currents ($\underline{J}_s$,$\underline{M}_s$) are introduced on the surface of the scatterer.

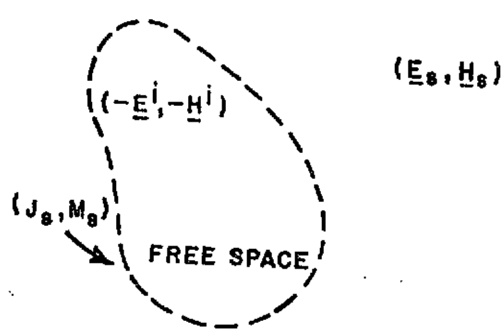

Fig. 1c--The exterior scattered field may be generated by ($\underline{J}_s$,$\underline{M}_s$) in free space.

With the scatterer replaced by free space, we have noted in Fig. 1b that the interior region has a null field. As shown in Fig. 2, we place an electric test source $\underline{J}_t$ in this region and find from the reciprocity theorem that

$$(5) \qquad \oint\!\!\oint_S (\underline{J}_s \cdot \underline{E}_t - \underline{M}_s \cdot \underline{H}_t)\, ds + \iiint (\underline{J}_i \cdot \underline{E}_t - \underline{M}_i \cdot \underline{H}_t)\, dv = 0$$

where $(\underline{E}_t, \underline{H}_t)$ is the free-space field of the test source. In words, Eq. (5) states that the interior test source has zero reaction with the other sources. This "zero-reaction theorem" was developed by Rumsey [13].

Equation (5) is the integral equation for the scattering problem, and our objective is to use this equation to determine the surface-current distributions $\underline{J}_s$ and $\underline{M}_s$. To accomplish this, we expand these functions in finite series so there will be a finite number N of unknown expansion constants. Next we obtain N simultaneous linear equations to permit a solution for these constants. One such equation is obtained from Eq. (5) each time we set up a new test source.

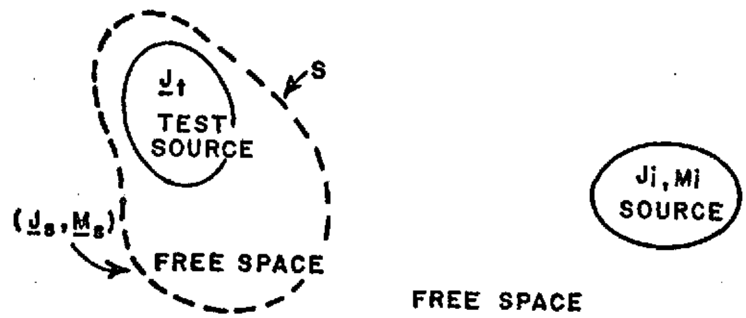

Fig. 2--An electric test source $\underline{J}_t$ is positioned in the interior of the scattering region.

The magnetic current $\underline{M}_s$ vanishes if the scatterer is a perfect conductor. We assume a finite conductivity and use the impedance boundary condition:

$$(6) \qquad \underline{M}_s = Z_s\, \underline{J}_s \times \hat{n}$$

where Z_s denotes the surface impedance defined by $\hat{n} \times \underline{E} = z_s\, \hat{n} \times \underline{H}$.

For three-dimensional problems involving arbitrary scatterers, $\underline{J}_S$ and $\underline{M}_S$ are functions only of the position on the surface of the scatterer. Eqs. (5) and (6) yield

$$(7) \qquad -\iint_S \underline{J}_S \cdot [\underline{E}_m - (\hat{n} \times \underline{H}_m) Z_S] \, ds = \iint \underline{J}_i \cdot \underline{E}_m \, ds - \iint \underline{M}_i \cdot \underline{H}_m \, ds$$

where $(\underline{E}_m, \underline{H}_m)$ denotes the free-space field of test-source m.

We represent the electric current distribution as follows:

$$(8) \qquad \underline{J}_S = \sum_{n=1}^{N} I_n \underline{J}_n$$

where the complex constants I_n are samples of the function J_S. The vector functions $\underline{J}_n$ are known as basis functions, subsectional bases, expansion functions or dipole modes. We employ expansion functions $\underline{J}_n$ and test sources $\underline{J}_m$ with unit current density at the terminals.

From Eqs. (7) and (8) we obtain the simultaneous linear equations

$$(9) \qquad \sum_{n=1}^{N} I_n C_{mn} = A_m \quad \text{with} \quad m = 1,2,3, \cdots N$$

where

$$(10) \qquad C_{mn} = -\iint_n \underline{J}_n \cdot [\underline{E}_m - (\hat{n} \times \underline{H}_m) Z_S] \, ds = -\iint_m \underline{J}_m \cdot \underline{E}_n \, ds$$

$$(11) \qquad A_m = \iint_i \underline{J}_i \cdot \underline{E}_m \, ds - \iint_i \underline{M}_i \cdot \underline{H}_m \, ds = \iint_m \underline{J}_m \cdot \underline{E}_i \, ds$$

In Eqs. (10) and (11) the integrations extend over the region where the integrand is non-zero. For example, region n is that portion of the surface S covered by the expansion function $\underline{J}_n$. Region m covers the interior test source $\underline{J}_m$. The reciprocity theorem relates the first and second integrals in Eq. (10). In the second integral, $\underline{E}_n$ is the free-space field generated by $\underline{J}_n$ and the associated magnetic current $\underline{M}_n$.

For computational speed and storage, it will be advantageous to have a symmetric impedance matrix C_{mn}. Furthermore, the test sources should be selected to yield a well-conditioned set of simultaneous linear equations. For these reasons, we employ test sources $\underline{J}_m$ of the same size, shape and functional form as the

expansion functions $\underline{J}_n$. Finally, we position the interior test sources a small distance δ from surface S and take the limiting form of the integrals as δ tends to zero.

The effect of a dielectric coating on a conducting body will now be considered. For simplicity, let the dielectric layer have the same permeability as free space. From the volume equivalence theorem of Rhodes [16] the dielectric coating may be replaced with free space and an equivalent electric current density

$$(12) \qquad \underline{J}_{eq} = j\omega(\varepsilon-\varepsilon_o)\ \underline{E}$$

where $\underline{E}$ denotes the electric field intensity in the dielectric and $\varepsilon = \varepsilon_r\varepsilon_o$ is the permittivity of the dielectric layer. From Eq. (12) the equivalent current $\underline{J}_{eq}$ vanishes outside the region of the dielectric coating.

Let $(\underline{E},\underline{H})$ denote the field generated by $(\underline{J}_i,\underline{M}_i)$ in the presence of a dielectric-coated conducting scatterer. Outside the scatterer, this field may also be generated by $(\underline{J}_i,\underline{M}_i)$, $(\underline{J}_s,\underline{M}_s)$ and $\underline{J}_{eq}$, radiating in free space. These sources, radiating in free space, generate a null field in the interior region of the conducting body. The surface currents $(\underline{J}_s,\underline{M}_s)$ are located on the surface of the conducting body and are related to the field $(\underline{E},\underline{H})$ by Eqs. (1) and (2).

For a coated conducting body, the reaction integral equation (Eq. (7)) is modified by replacing $\underline{J}_i$ with $\underline{J}_i + \underline{J}_{eq}$. The current density $\underline{J}_{eq}$ may be regarded as an additional source which plays much the same role as the impressed source $\underline{J}_i$. However, $\underline{J}_{eq}$ is an unknown quantity because $\underline{E}$ is unknown. If the dielectric coating is thin, Maxwell's equations and boundary conditions can be employed to express the polarization-current distribution $\underline{J}_{eq}$ in terms of the surface-current density on the conducting surface. Therefore, $\underline{J}_{eq}$ may be regarded as a dependent unknown function because it is simply related to $\underline{J}_s$.

The polarization-current distribution is expanded as follows:

$$(13) \qquad \underline{J}_{eq} = \sum_{n=1}^{N} I_n\ \underline{\mathcal{J}}_n$$

where the $\underline{\mathcal{J}}_n$ are functions simply related to $\underline{J}_n$. Thus, for a coated conducting body, each expansion mode $\underline{J}_n$ in Eq. (8) has associated with it a polarization current $\underline{\mathcal{J}}_n$, and the reaction C_{mn} between the electric test source m and the expansion mode n has an additional term given by

(14) $\Delta C_{mn} = -\iint_n \underline{\mathcal{J}}_n \cdot \underline{E}_m \, ds$

where the integration extends through the dielectric coating in the range of the expansion mode n. The functions $\underline{J}_n$ are defined over a surface, while the $\underline{\mathcal{J}}_n$ are defined in a volumetric region.

It may be noted that in the reaction formulation, the effects of a dielectric coating are accounted for entirely through a modification of the square reaction matrix C_{mn}. This modification influences the current distribution, field patterns and scattering properties.

The following chapter discusses the electric sources which are employed as test sources and expansion modes for the current distribution on the conducting surface.

CHAPTER III
ELECTRIC TEST SOURCES AND EXPANSION MODES

In the preceding chapter, the reaction integral equation is reduced to a matrix equation via the moment method in two steps. First the unknown current distribution is expanded with basis functions. Test sources with the same functional form are then employed to perform the reaction tests. In the following sections three types of test sources employed in this dissertation are discussed.

A. LONGITUDINAL STRIP SOURCES

For an infinitely long cylinder with transverse magnetic incident wave, the electric current density induced on the conducting surface is in the longitudinal direction. Thus a suitable choice of the bases is the longitudinal strip source.

Consider the "strip source" illustrated in Fig. 3. This source is an electric surface-current distribution $\underline{J} = \hat{z}\, J(x)$ located on the xz plane. This source has width h and infinite length and radiates in free space. For the strip source shown in Fig. 3, the fields are given as follows:

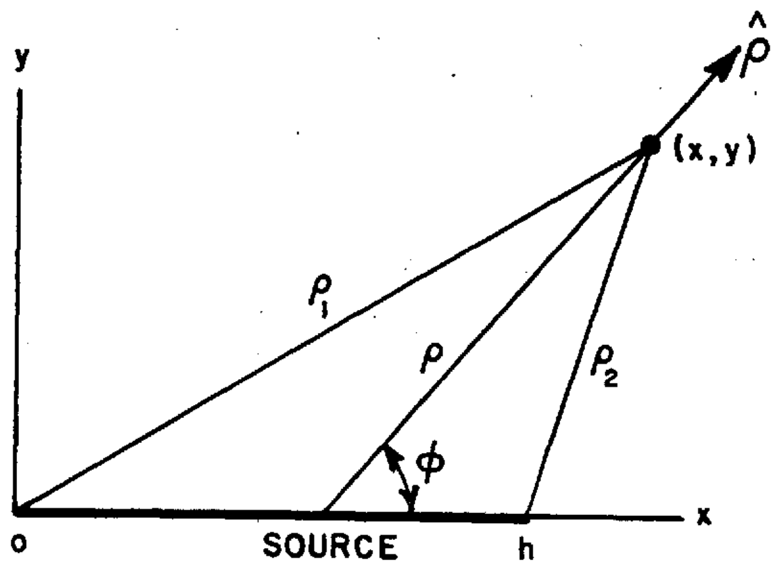

Fig. 3--An electric strip source and the coordinate system.

(15) $$\underline{E} = \frac{k\eta}{4} \int_0^h \underline{J}\, H_0^{(2)}(k\rho)\, dx'$$

(16) $\underline{H} = -\frac{jk}{4}\int_0^h \underline{J} \times \hat{\rho}\, H_1^{(2)}(k\rho)\, dx'$

where

(17) $\rho = \sqrt{(x-x')^2 + y^2}$

(18) $k = \omega\sqrt{\mu\varepsilon}$

(19) $\eta = \sqrt{\mu/\varepsilon}$

If the electric strip source has a uniform distribution,

(20) $\underline{J}(x) = \hat{z}$

Eqs. (15) and (16) can be written as

(21) $E_z = -\frac{k\eta}{4}\int_0^h H_o^{(2)}(k\rho)\, dx'$

(22) $\underline{H} = -\frac{jk}{4}\int_0^h \hat{z} \times \hat{\rho}\, H_1^{(2)}(k\rho)\, dx'$.

B. TRANSVERSE STRIP SOURCES [9]

In the case of a conducting cylinder illuminated by a transverse-electric incident wave, the electric current induced on the surface is in the transverse direction. Therefore, transverse electric sources are the natural choice for the induced current density.

Again consider the "strip source" illustrated in Fig. 3. But this time the source is an electric surface-current distribution $\underline{J} = \hat{x}\, J(x)$ located on the xz plane. For this transverse electric source the free space fields are

(23) $\underline{E} = -\frac{k\eta}{4}\int_0^h \underline{J}\, H_o^{(2)}(k\rho)\, dx' + \frac{\eta}{4}\int_0^h \hat{\rho}\, J'\, H_1^{(2)}(k\rho)\, dx'$

(24) $\underline{H} = -\frac{jk}{4}\int_0^h \underline{J} \times \hat{\rho}\, H_1^{(2)}(k\rho)\, dx'$

where

(25) $\quad J' = dJ/dx'$

For most current functions J(x), the field integrals must be evaluated with infinite-series expansions or numerical integration procedures. For the sinusoidal current distribution, however, E_x is obtained in simple closed form [9]. Thus, if

(26) $\quad \underline{J}(x) = \hat{x}[I_1 \sin(kh-kx) + I_2 \sin(kx)]/\sin(kh)$

then

(27) $$E_x = \frac{\eta}{4\sin(kh)}\left[I_1 H_0^{(2)}(k\rho_1)\cos(kh) - I_1 H_0^{(2)}(k\rho_2)\right] + \left[I_2 H_0^{(2)}(k\rho_2)\cos(kh) - I_2 H_0^{(2)}(k\rho_1)\right]$$

where I_1 and I_2 represent J(0) and J(h), respectively, and ρ_1, ρ_2 are illustrated in Fig. 3.

For the purpose of representing a continuous surface-current distribution on a conducting cylinder, it is useful to define a strip dipole which is comprised of two strip monopoles. Fig. 4a illustrates a planar strip dipole. This dipole lies on the xz plane and has infinite length in the z direction. The surface-current density is

(28) $$\underline{J} = \hat{x}\frac{\sin k(x-x_1)}{\sin k(x_2-x_1)} \qquad \text{for} \qquad x_1 \le x \le x_2$$

(29) $$\underline{J} = \hat{x}\frac{\sin k(x_3-x)}{\sin k(x_3-x_2)} \qquad \text{for} \qquad x_2 \le x \le x_3$$

As indicated in Fig. 4b, the current density vanishes at the edges x_1 and x_3, is continuous across the terminals at x_2 and has a slope discontinuity at x_2.

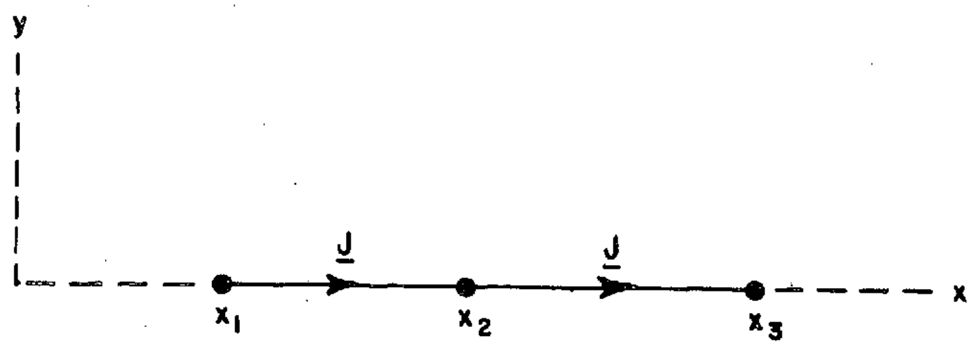

Fig. 4a--A planar strip dipole with edges at x_1 and x_3 and terminals at x_2.

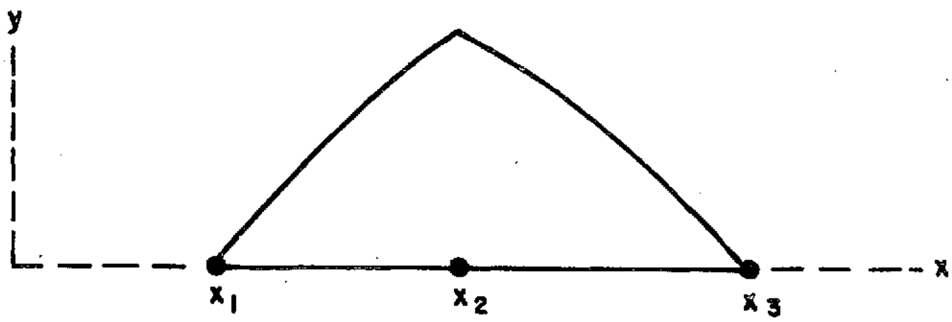

Fig. 4b--The current-density distribution $\underline{J}$ on the sinusoidal strip dipole.

Fig. 5 illustrates a strip V-dipole. Distance along the dipole arms is measured by the coordinates s and t with origin at the terminals 0. The surface-current density is

$$(30) \qquad \underline{J} = -\hat{s}\,\frac{\sin k(s_1-s)}{\sin ks_1} \qquad \text{on} \quad \text{arm } s$$

$$(31) \qquad \underline{J} = \hat{t}\,\frac{\sin k(t_1-t)}{\sin kt_1} \qquad \text{on} \quad \text{arm } t,$$

where the unit vectors $\hat{s}$ and $\hat{t}$ are perpendicular to the z-axis. Thus, the current density vanishes at edges s_1 and t_1 and has unit value at the terminals 0. The edges are parallel with the z-axis. If the wedge angle ψ is adjusted to 180 degrees, the V-dipole in Fig. 5 reduces to the planar dipole in Fig. 4.

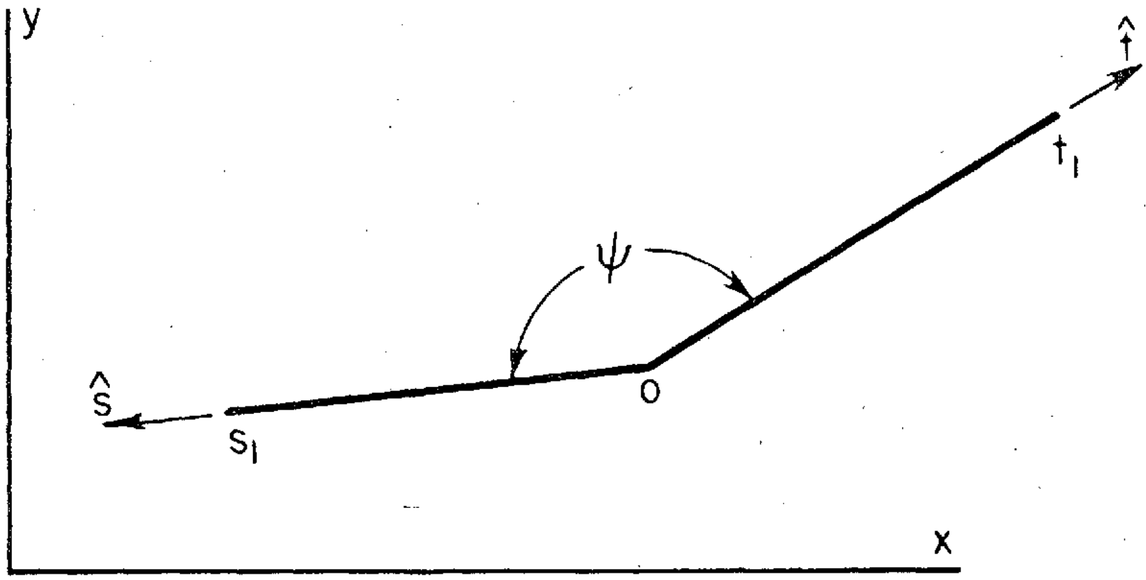

Fig. 5--Nonplanar strip dipole with edges at s_1 and t_1 and terminals at 0.

C. RECTANGULAR SURFACE SOURCES

Consider a "surface monopole" radiating in free space as shown in Fig. 6. This source is an electric surface-current density $\underline{J} = \hat{z}\, J(z)$ located on the yz plane. This source has height a/2 and width b. The surface-current density is related to the current by $\underline{I} = b\, \underline{J}$. For the electric surface monopole illustrated in Fig. 6, the fields are

$$(32) \qquad \underline{E} = \frac{-j}{\omega\varepsilon}\left[k^2\int_0^{a/2}\int_0^b \underline{J}\, G_o(kR)\, dy'\, dz' + \nabla\int_0^{a/2}\int_0^b J' G_o(kR) dy' dz'\right]$$

$$(33) \qquad \underline{H} = \nabla \times \int_0^{a/2}\int_0^b \underline{J}\, G_o(kR)\, dy'\, dz'$$

where

$$(34) \qquad R = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$$

$$(35) \qquad J' = dJ/dz'$$

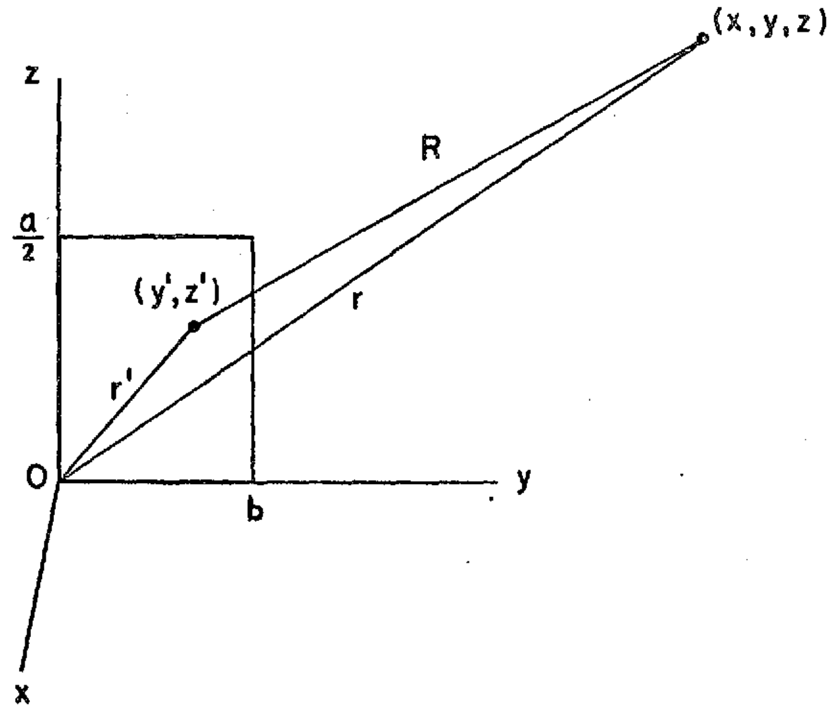

Fig. 6--An electric surface monopole and the coordinate system.

$$(36)\qquad G_o(kR) = e^{-jkR}/4\pi R$$

Numerical integration techniques must be employed to perform the field integrals.

A planar sinusoidal dipole source located on the yz plane as shown in Fig. 7a will be considered. This source is an electric surface-current density with height a and width b. The surface-current density is given by

$$(37)\qquad \underline{J} = \hat{z}\cos\left(\frac{(z-z_2)\pi}{2(z_2-z_1)}\right) \qquad \text{for} \quad z_1 \le z \le z_2$$

$$(38)\qquad \underline{J} = \hat{z}\cos\left(\frac{(z-z_2)\pi}{2(z_3-z_2)}\right) \qquad \text{for} \quad z_2 \le z \le z_3$$

As illustrated in Fig. 7b and 7c, the current density vanishes at the edges $z = z_1$ and $z = z_3$, and is uniformly distributed in the

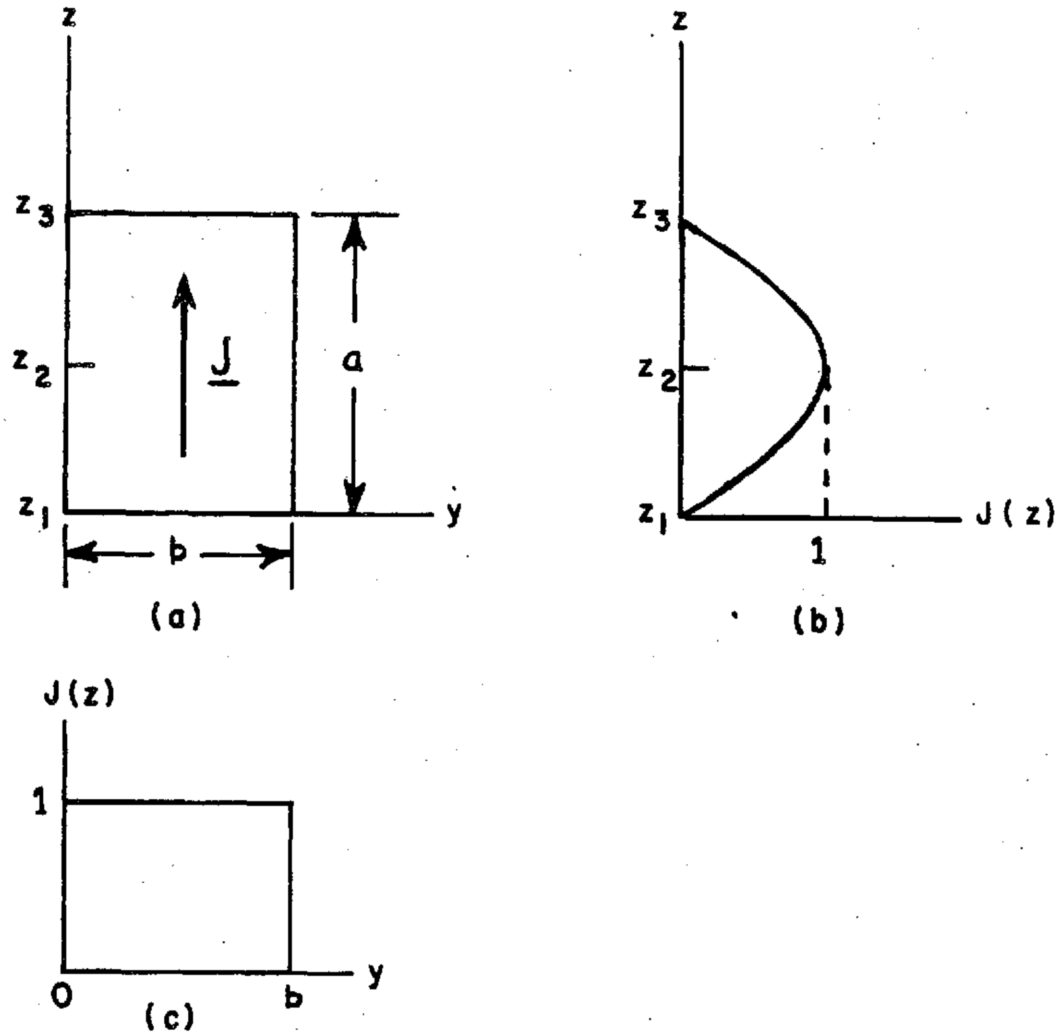

Fig. 7--An electric surface dipole and its current-density distribution.

transverse direction. The surface-current density and its slope are continuous across the terminals at $z = z_2$ for $z_2 - z_1 = z_3 - z_2 = a/2$.

Fig. 8 illustrates a surface V-dipole. Distance along the dipole arms is measured by the coordinates s and t with origin at the terminal O. The surface-current density for this electric source is

$$(39) \qquad \underline{J} = -\hat{s}\cos\left(\frac{\pi s}{2s_1}\right) \qquad \text{on arm s}$$

(40) $\quad \underline{J} = \hat{t} \cos\left(\frac{\pi t}{2t_1}\right) \qquad$ on arm t .

When the wedge angle ψ is adjusted to 180 degrees, the V-dipole in Fig.8 reduces to the planar surface dipole in Fig. 7a.

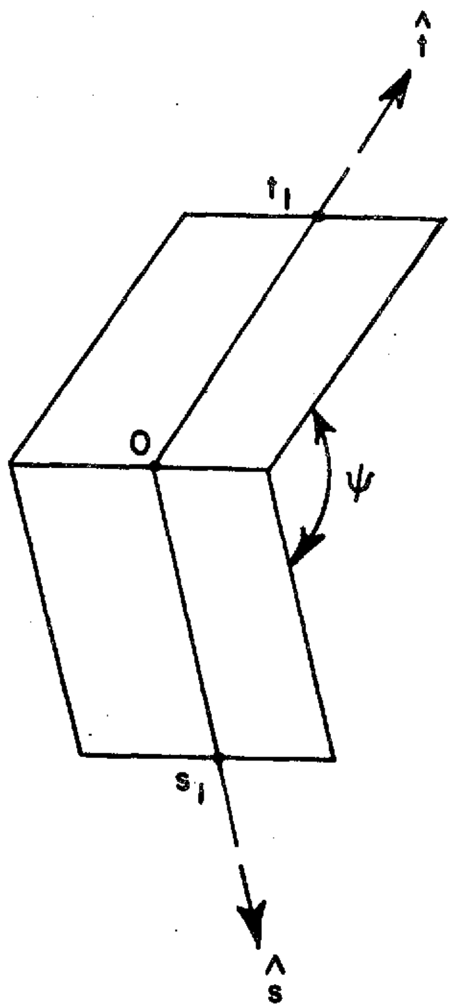

Fig. 8--A nonplanar surface dipole with edges at s_1 and t_1 and terminal at 0.

The electric sources defined in the previous sections are all hypothetical sources. The current density on a conducting strip is neither uniform nor sinusoidal and the current density induced on a rectangular plate is not sinusoidal. The relevance of these sources will now be explained. The electric current distributions (Eqs. (20),

(30), (31), (39), and (40)) will be used as the basis functions (Eq. (8)) for expanding the unknown current distribution induced on conducting surfaces for various problems. Furthermore, test sources with the same size, shape, and functional form as the expansion functions will be employed with the reaction concept to solve the integral equation.

By superposition, the field scattered by a conducting body may be regarded as the sum of the fields radiated by the mode currents:

$$(41) \qquad \underline{E}^s = \sum_{n=1}^{N} I_n \, \underline{e}_n$$

$$(42) \qquad \underline{H}^s = \sum_{n=1}^{N} I_n \, \underline{h}_n$$

where $(\underline{e}_n, \underline{h}_n)$ is the free space field generated by the mode current n.

CHAPTER IV
THE IMPEDANCE MATRIX

From the viewpoint of reaction, the complex number C_{mn} in Eq. (10) represents the reaction between the sources m and n. The reaction between two electric sources m and n is:

$$(43) \qquad C_{mn} = -\iiint_m \underline{E}_n \cdot \underline{J}_m \, dv$$

Although the electric sources defined in Chapter III are hypothetical, it is useful to define self impedance with the induced-emf formulation [13]:

$$(44) \qquad Z_{mm} = \frac{V_{mm}}{I_{mm}} = \frac{C_{mm}}{I_{mm}^2}$$

From Eqs. (43) and (44):

$$(45) \qquad Z_{mm} = \frac{-1}{I_{mm}^2} \iiint_m \underline{E}_m \cdot \underline{J}_m \, dv$$

where $\underline{J}_m$ is the current density of source m and $\underline{E}_m$ is the free-space electric field. The self impedance of the longitudinal strip source (Eq. (20)), the transverse strip dipole [9] (Eq. (30) and Eq. (31)), and the rectangular surface dipole (Eqs. (39) and (40)), as a function of size, are listed in Table I, II, and III, respectively.

The mutual impedance between two sources is defined by

$$(46) \qquad Z_{mn} = \frac{-1}{I_{mm} \, I_{nn}} \iiint_m \underline{E}_n \cdot \underline{J}_m \, dv.$$

Tables IV, V and VI list the mutual impedance between two longitudinal strip sources, transverse strip sources [9], and rectangular surface dipoles, respectively.

TABLE I

Self Impedance of Longitudinal Strip Source shown in Fig. 9.

h/λ	R_{11}	X_{11}
0.1	582.38	771.74
0.2	554.63	488.50
0.3	511.90	317.22
0.4	458.70	199.36
0.5	400.65	118.30

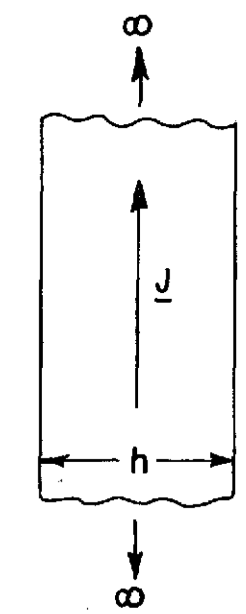

Fig. 9--Strip Source with $\underline{J} = \hat{z}$.

TABLE II

Self Impedance of Center-Fed Strip-Dipole
shown in Fig. 10

$s_1 = t_1 = h$

ψ	$h/\lambda = 0.05$	$h/\lambda = 0.10$	$h/\lambda = 0.15$	$h/\lambda = 0.20$
45°	0.11 -j 14.1	0.47 -j 13.4	1.20 -j 12.1	2.53 -j 10.4
90°	0.38 -j 20.9	1.59 -j 19.3	3.94 -j 17.1	8.10 -j 14.0
135°	0.64 -j 24.3	2.68 -j 22.2	6.49 -j 19.5	12.95 -j 16.4
180°	0.75 -j 25.3	3.11 -j 23.0	7.50 -j 20.3	14.77 -j 17.4

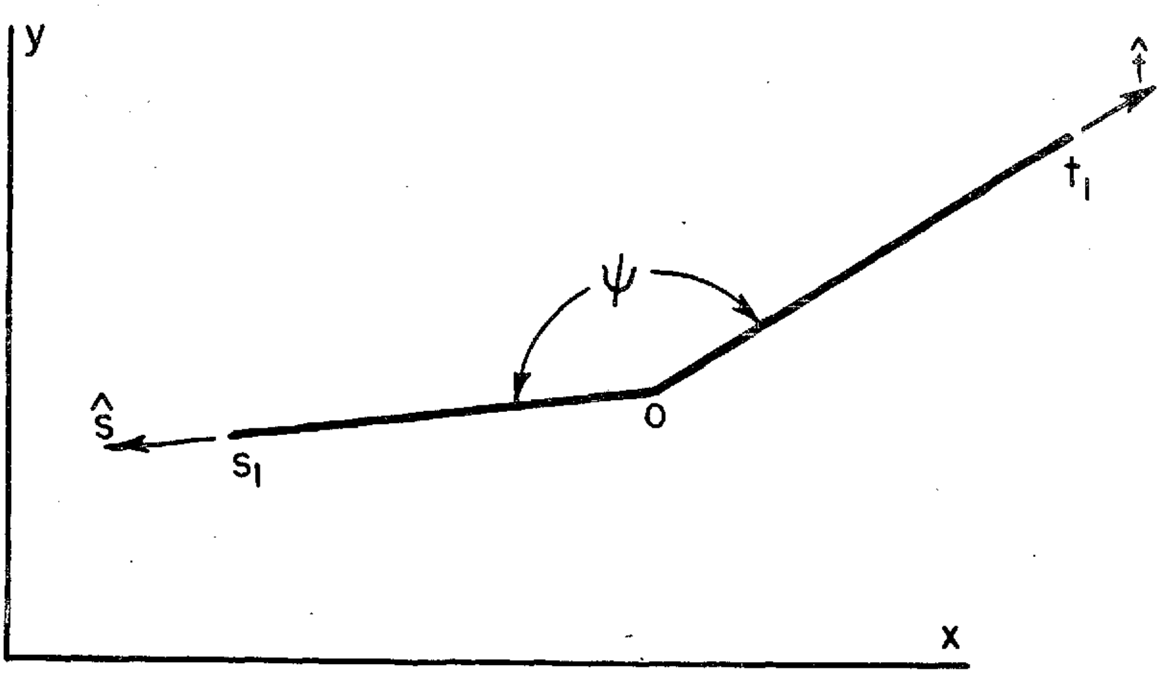

Fig. 10--Nonplanar strip dipole with edges at s_1 and t_1 and terminals at 0.

TABLE III

Self Impedance of Center-Fed Planar Surface-Dipole shown in Fig. 11

a/λ	R_{11}	X_{11}
0.2	11.48	-69.76
0.3	23.98	-35.26
0.4	38.80	-15.36
0.5	52.98	- 4.52

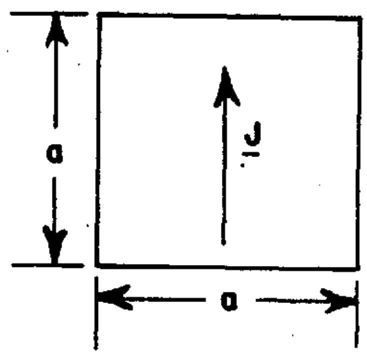

Fig. 11--Surface Dipole with $\underline{J} = \hat{z}\cos(\pi z/a)$

TABLE IV

Mutual Impedance of Coplanar Strip Sources shown in Fig. 12

$h_1 = h_2 = 0.1/\lambda$

d/λ	R_{12}	X_{12}
0.0	582.38	771.74
0.1	526.74	205.89
0.2	376.25	-140.97
0.3	171.77	-283.09
0.4	-27.43	-283.44
0.5	-172.11	-190.34

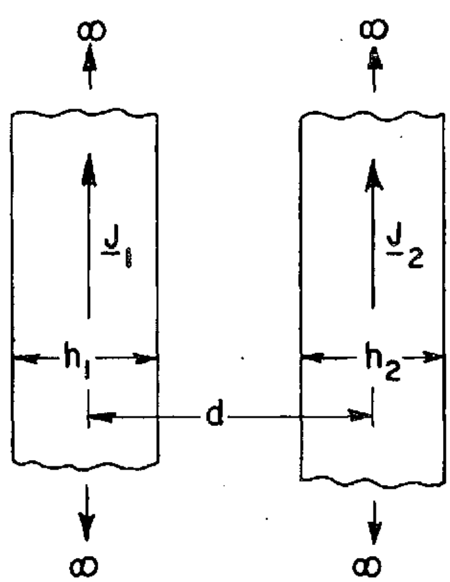

Fig. 12--Coupled Coplanar Strip Sources with $\underline{J}_1 = \underline{J}_2 = \hat{z}$

TABLE V

Mutual Impedance of Center-Fed Planar Strip-Dipoles shown in Fig. 13

Segment length: $h/\lambda = 0.1$
Distance between midpoints: $\rho/\lambda = 0.3$

ϕ	$\beta = 0°$	$\beta = 45°$	$\beta = 90°$	$\beta = 135°$
0°	1.94 +j 0.81	1.36 +j 0.39	0.00 +j 0.00	-1.36 -j 0.39
30°	1.42 -j 0.58	1.62 +j 1.07	0.87 +j 1.80	-0.38 +j 1.73
60°	0.39 -j 2.68	0.90 -j 0.66	0.87 +j 1.80	0.34 +j 3.03
90°	-.13 -j 3.53	-.08 -j 2.57	0.00 +j 0.00	0.08 +j 2.57

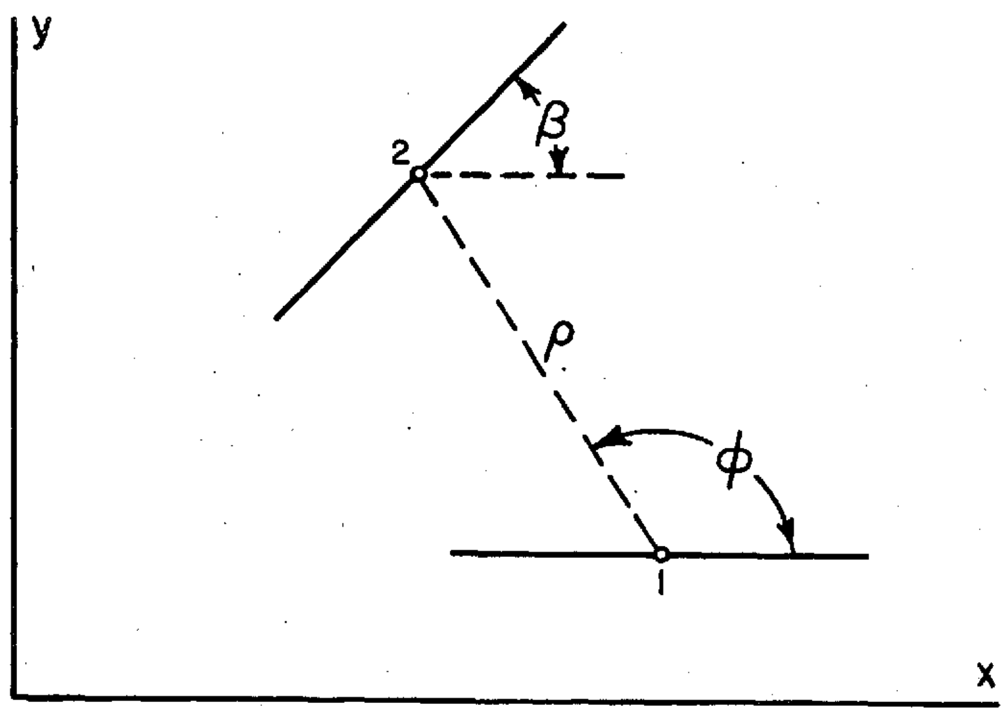

Fig. 13--Coupled strip dipoles

TABLE VI

Mutual Impedance of Center-Fed Coplanar Surface-Dipoles
shown in Fig. 14

$a/\lambda = 0.5$ $b/\lambda = 0.25$

0.75	1.076 -j 7.913	-2.925 -j 5.950	-5.968 +j 1.761	0.8871 +j 7.019
0.50	24.43 +j 5.997	10.04 -j 9.321	-9.568 -j 6.65	-7.194 +j 8.971
0.25	53.10 +j 55.87	29.54 -j 9.070	-9.468 -j 19.33	-15.74 +j 6.426
0.0	67.09 +j 13.23	39.07 -j 22.49	-8.659 -j 26.75	-19.32 +j 4.263
s_z/λ \ s_y/λ	0.0	0.25	0.50	0.75

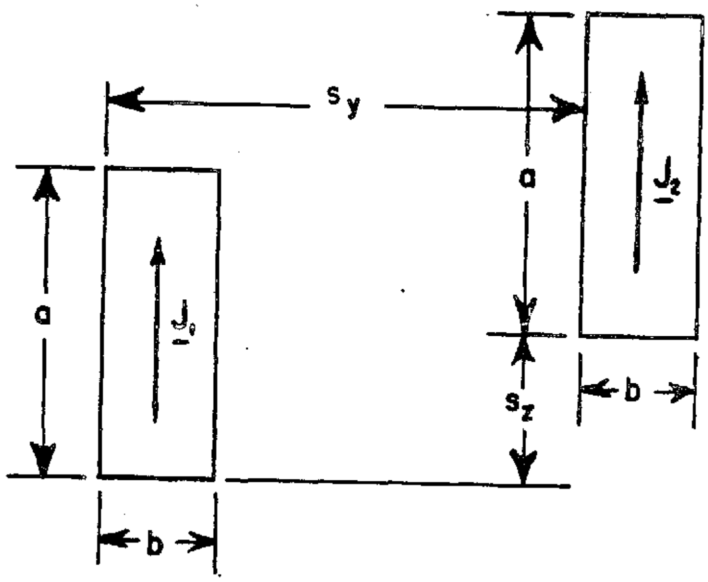

Fig. 14--Coupled surface dipoles

CHAPTER V
THE EXCITATION COLUMN

The complex quantities A_m in Eq. (9) form the excitation column in the matrix equation $C_{mn} I_n = A_m$. Physically, A_m is the reaction between the impressed source and the test source m. These reactions are independent of the surface impedance or the dielectric coating. For arbitrary impressed sources which generate $\underline{E}_i$ in free space, Eq. (11) gives

$$(47) \qquad A_m = \iiint_m \underline{J}_m \cdot \underline{E}_i \, dv \ .$$

The above expressions require numerical integration over test source m.

A. PLANE WAVE ILLUMINATION (TWO-DIMENSIONAL TM CASE)

If an infinitely long, electric line source, parallel with the z axis, is located at a great distance from the conducting cylinder, the incident field $(\underline{E}_i, \underline{H}_i)$ may be regarded as a plane wave with

$$(48) \qquad \underline{E}_i = \hat{z} E_0 e^{jk(x \cos \phi_i + y \sin \phi_i)}$$

where ϕ_i is the angular coordinate of the source, and E_0 is the incident electric field intensity at the origin. Fig. 15 illustrates an incident plane wave illuminating a longitudinal strip source with unit current density in the z direction. The integration in Eq. (47) is readily performed to yield

$$(49) \qquad A_m = E_0 \frac{e^{j\psi_2} - e^{j\psi_1}}{jk \cos(\alpha - \phi_i)}$$

where $\psi_j = k(x_j \cos \phi_i + y_j \sin \phi_i)$ and α is the angle between the positive x axis and the vector directed from point 1 to point 2.

B. PLANE WAVE ILLUMINATION (TWO-DIMENSIONAL TE CASE)

Consider a magnetic line source, parallel with the z axis and located at a great distance away, illuminating a strip dipole with sinusoidal electric current density (Eqs. (30) and (31)) flow in the direction from 1 to 2 and from 2 to 3 as shown in Fig. 16. The incident electric field may be regarded as a plane wave with

$$(50) \qquad \underline{E}_i = -\hat{\phi}_i \eta H_o e^{jk(x \cos \phi_i + y \sin \phi_i)}$$

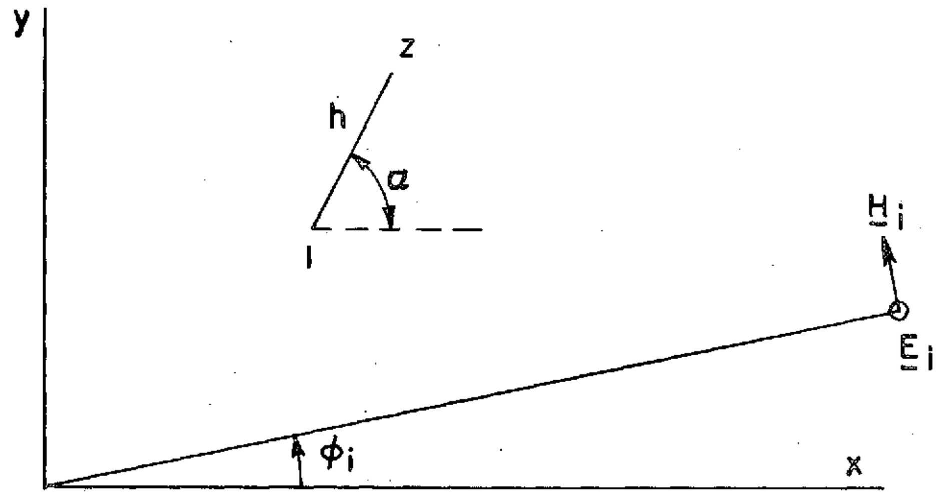

Fig. 15--A plane wave illuminates a strip source

In this case, Eq. (47) is readily evaluated to yield

$$(51)\qquad A_m = -\eta\, H_o \frac{\left[e^{j\psi_1} - (\cos kh_1 - j\cos(\alpha_1 - \phi_i)\sin kh_1)e^{j\psi_2}\right]}{k \sin kh_1 \sin(\alpha_1 - \phi_i)} + \eta\, H_o \frac{\left[e^{j\psi_3} - (\cos kh_2 - j\cos(\alpha_2 - \phi_i)\sin kh_2)e^{j\psi_2}\right]}{k \sin kh_2 \sin(\alpha_2 - \phi_i)}$$

where h_1 and h_2 are the dipole segment lengths. The angle between the positive x axis and the vector directed to the terminals from point 1 is denoted α_1. Similarly α_2 is the angle of the vector directed to the terminals from point 3.

C. PLANE WAVE ILLUMINATION (THREE-DIMENSIONAL CASE)

If an impressed source is located at a great distance away from a surface dipole with current distribution $\underline{J} = \hat{z}' \cos(\pi z'/a)$ as shown in Fig. 17, the incident field may be regarded as a plane wave with

$$(52)\qquad \underline{E}_i = \underline{E}_o\, e^{jk(x'\sin\theta_i' \cos\phi_i' + y'\sin\theta_i' \sin\phi_i' + z'\cos\theta_i')} .$$

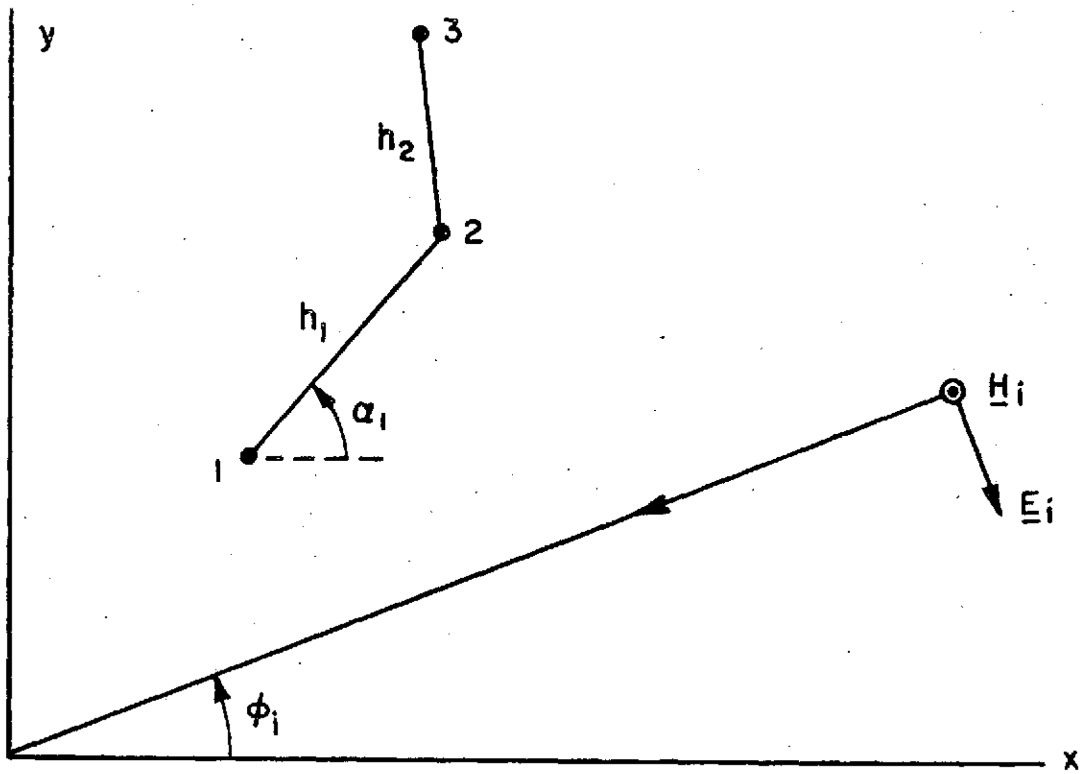

Fig. 16--A plane wave illuminates an electric strip dipole.

From Eqs. (52) and (47),

$$A_m = (\underline{E}_0 \cdot \hat{z}')2\pi ab \frac{\sin(X_i)\cos(Y_i/2)}{X_i(Y_i^2-\pi^2)} \tag{53}$$

where $X_i = 0.5\ kb \sin\theta_i' \sin\phi_i'$

$Y_i = ka \cos\theta_i'$.

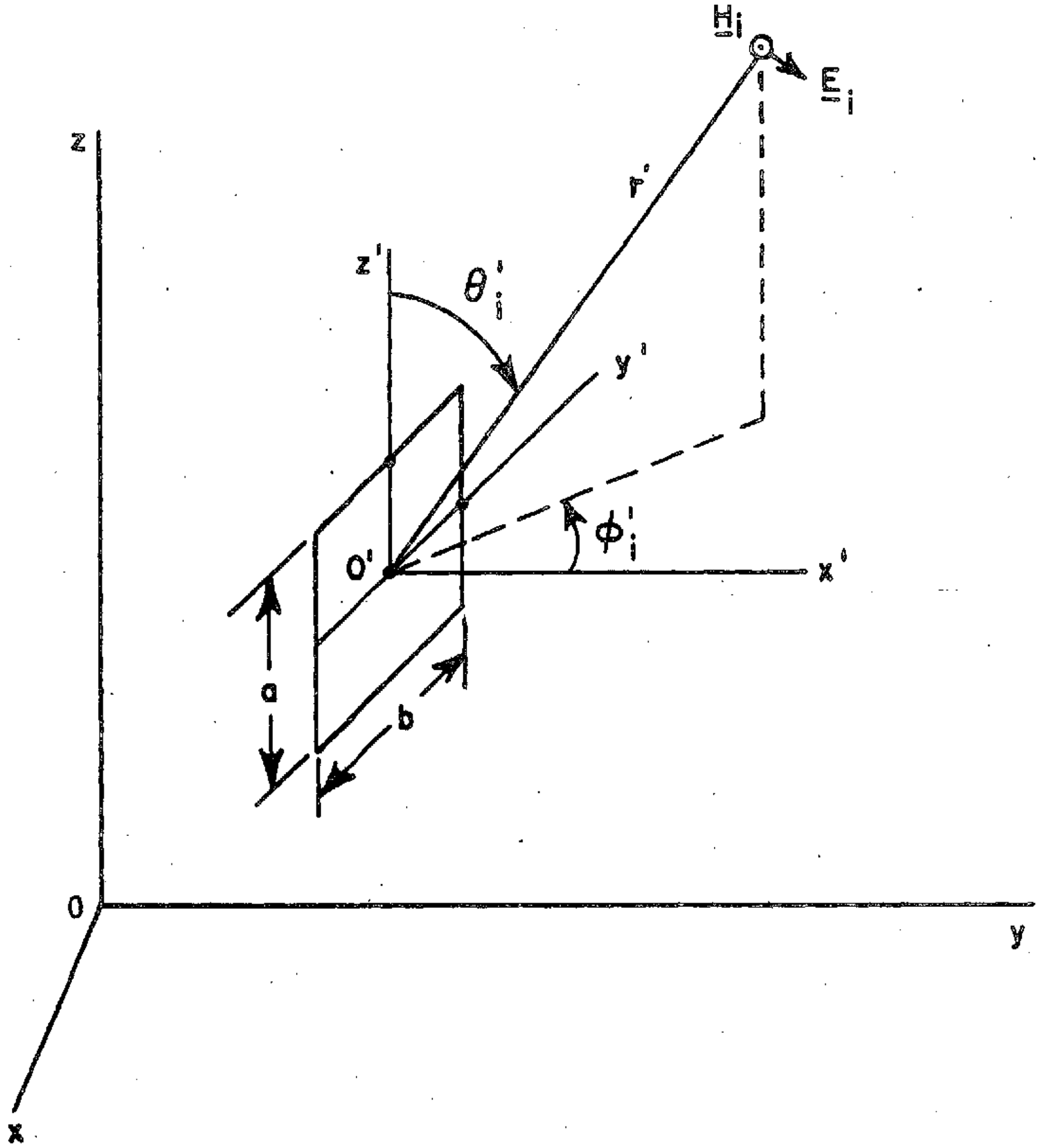

Fig. 17--A plane wave ($\underline{E}_i$,$\underline{H}_i$) illuminates an electric surface dipole.

CHAPTER VI
APPLICATIONS

In the preceding chapters, attention has been directed to the general theory of the reaction integral formulation for scattering problems. Suitable bases and test sources have been defined and the mutual impedance between them has been evaluated. In the following, scattering from dielectric-coated cylinders with transverse magnetic incident wave will first be investigated.

A. CYLINDERS WITH THIN DIELECTRIC COATING (TM CASE)

Consider a dielectric-coated conducting cylinder illuminated by an electric line source. Let $(\underline{J}_s, \underline{M}_s)$ denote the surface-current density induced on the conducting surface, and $\underline{J}_{eq}$ denotes the polarization-current density induced in the dielectric layer. The first step in the reaction-Galerkin approach is to approximate the cylinder by a polygon cylinder with N segments. This is accomplished by fitting the cylinder with segments such that the perimeter of the polygon cylinder is equal to that of the original cylinder.

Fig. 18a illustrates a dielectric-coated, conducting polygon cylinder illuminated by a parallel electric line source $J_i \hat{z}$. Let I_n denote the current density $\underline{J}_s$ on segment n of the polygon cylinder. Each modal current $\underline{J}_n$ has a uniform current distribution as in Eq. (20). Now one represents $\underline{J}_s$ as the superposition of the N modal currents with weighting I_n. This gives a piecewise uniform expansion for $\underline{J}_s$ with N unknown constants. The expansion for $\underline{J}_{eq}$ will be considered next.

For a transverse magnetic source such as the electric line source $J_i \hat{z}$ shown in Fig. 18a, the electric field has only a $\hat{z}$-component. For a conducting cylinder with thin dielectric coating, Maxwell's equations and the boundary conditions can be employed to obtain a suitable approximation for the electric field in the dielectric layer (Appendix):

$$(54) \qquad \underline{E} = \frac{-k_1 \sin(k_1\zeta)\underline{J}_s}{j\omega\varepsilon} \qquad \text{(TM case)}$$

where $\underline{J}_s$ is the $\hat{z}$-directed surface-current distribution on the conducting surface and ε is the permittivity of the layer. Distance measured normally outward from the conducting surface is denoted by the coordinate ζ, and k_1 is the propagation constant in the dielectric.

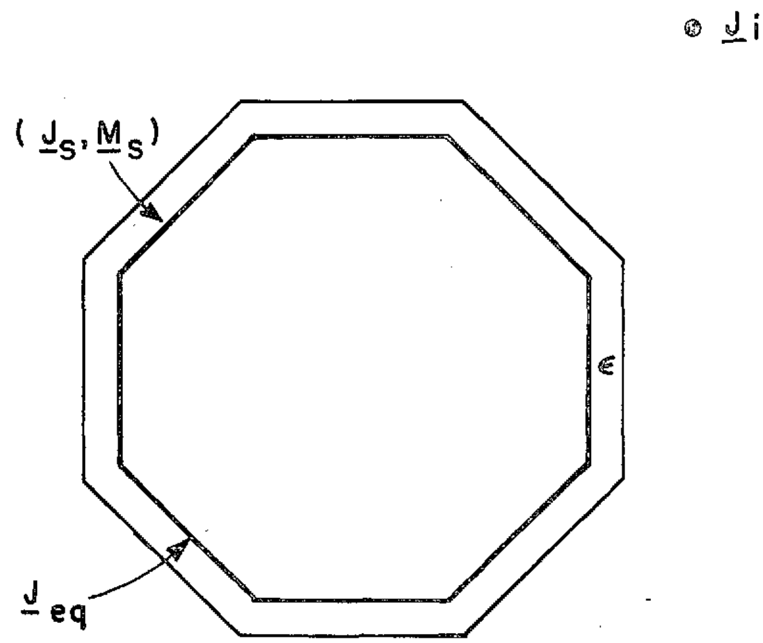

Fig. 18a--Dielectric-coated, conducting polygon cylinder illuminated by a parallel electric line source.

From Eq. (12) and (54), the polarization-current density can be expressed in terms of the surface-current density as follows.

$$\underline{J}_{eq} = -\left(\frac{\varepsilon-\varepsilon_0}{\varepsilon}\right) k_1 \sin(k_1\zeta)\underline{J}_s \qquad \text{(TM case)} \tag{55}$$

Since the surface-current distribution $\underline{J}_s$ has been expanded with rectangular-pulse bases $\underline{J}_n$, one obtains a dependent expansion for the polarization-current density

$$\underline{J}_{eq} = \sum_{n=1}^{N} I_n \tilde{\underline{J}}_n \tag{56}$$

where

$$(57) \qquad \tilde{\underline{J}}_n = -\left(\frac{\varepsilon-\varepsilon_0}{\varepsilon}\right) k_1 \sin(k_1\zeta) \underline{J}_n \qquad \text{(TM case)}$$

The magnetic current $\underline{M}_s$ vanishes if the cylinder is a perfect conductor. If the cylinder has finite conductivity and the impedance boundary condition (Eq. (6)) is employed, then the magnetic surface-current density can be expanded as

$$(58) \qquad \underline{M}_s = \sum_{n=1}^{N} I_n \underline{M}_n$$

where

$$(59) \qquad \underline{M}_n = z_s \underline{J}_n \times \hat{n} \qquad \text{(TM case)}$$

and $\hat{n}$ is the unit normal vector directed into the source region.

From the above discussions, the original problem illustrated in Fig. 18a can be replaced by its equivalent electromagnetic model, in that the dielectric-coated polygon cylinder is represented by an array of N hypothetical sources radiating in free space as shown in Fig. 18b. Each source is a modal current distribution $\underline{S}_n$, which is a collection of mode currents $\underline{J}_n$, $\tilde{\underline{J}}_n$ and $\underline{M}_n$, with weighting I_n.

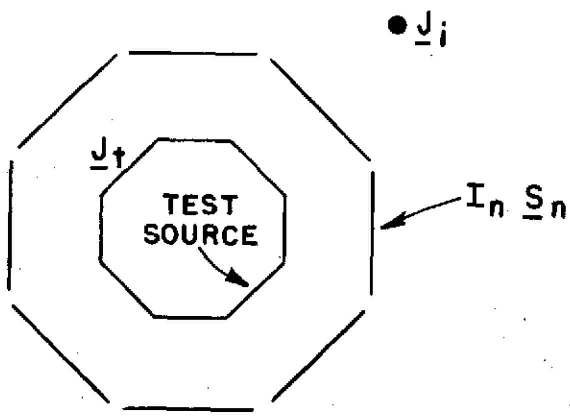

Fig. 18b--Electromagnetic model of the problem shown in Fig. 18a.

Based on the zero-reaction concept discussed in Chapter II, if one places an electric test source in the interior region, as illustrated in Fig. 18b, this interior test source has zero reaction with the other sources. To determine N current samples, one makes N independent reaction tests. This procedure generates a system of N simultaneous linear equations. Numerical solution of this system yields a stationary result [17] for the samples of the current distribution.

Let C_{mn} denote the reaction between test-source m and mode current $\underline{S}_n$ in Fig. 18b. The reaction between the test-source m and the impressed source $\underline{J}_i$ plus the reaction between the test-source m and all the mode currents,

$$\sum_{n=1}^{N} \underline{S}_n$$

must vanish, leading again to Eq. (9) in which the reaction C_{mn} is given by Eq. (10) plus an additional term ΔC_{mn} given by Eq. (14).

B. CYLINDERS WITH THIN DIELECTRIC COATING (TE CASE)

Consider a dielectric-coated, conducting polygon cylinder illuminated by a parallel magnetic line source $M_i\ \hat{z}$, as shown in Fig. 19a. The induced surface-current density $\underline{J}_s$ flows in the direction transverse to z axis. Let I_n denote the current density at the corners of the polygon, and let one define N strip dipole-mode currents on the conducting surface. Mode 1 extends from point N to point 2, Mode 2 extends from point 1 to point 3, and so on. Each mode $\underline{J}_n$ has a sinusoidal current distribution and unit terminal current density defined by Eqs. (30) and (31). Now one represents $\underline{J}_s$ as the superposition of the N overlapping dipole-mode currents with weighting I_n. This gives a piecewise-sinusoidal expansion for $\underline{J}_s$ with N unknown constants.

For the TE case, the electric field in the thin dielectric coating is essentially normal to the conducting surface, and can be determined from the charge density distributed on the conducting surface. Via Maxwell's equations and the boundary conditions, a suitable approximation for the electric field in the dielectric region (Appendix) is

$$(60) \qquad \underline{E} = \frac{1}{j\omega\varepsilon} \cos(k_\zeta \zeta)\ (\hat{z} \times \underline{J}_s') \qquad \text{(TE CASE)}$$

where $k_\zeta = k\sqrt{\varepsilon_r - 1}$ and ε_r is the relative permittivity. Coordinate ζ denotes the distance measured normally outward from the conducting surface. $\underline{J}_s'$ is the derivative of the surface-current density. From Eqs. (12) and (60), one obtains a suitable expansion for the polarization-current density inside the thin dielectric layer coated on a conducting cylinder as follows:

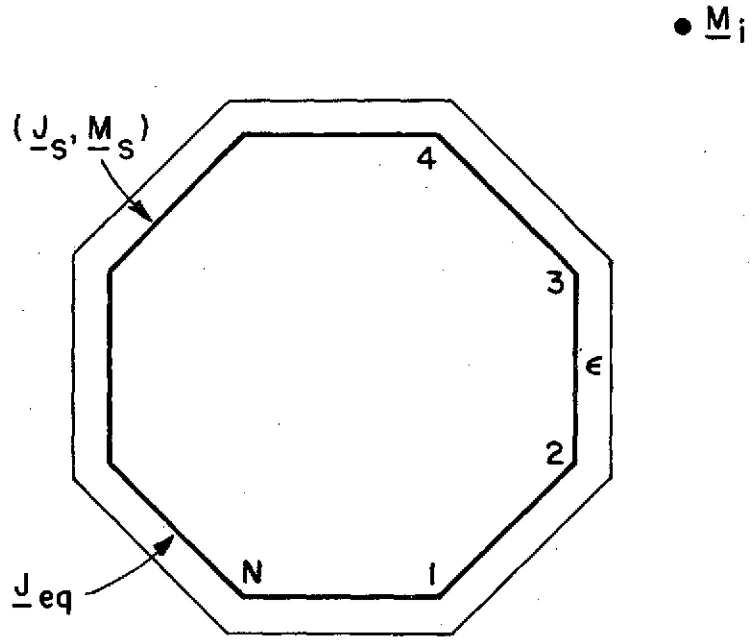

Fig. 19a--Dielectric-coated, conducting polygon cylinder illuminated by a parallel magnetic line source.

$$(61) \qquad \underline{J}_{eq} = \sum_{n=1}^{N} I_n \underline{\mathcal{J}}_n$$

where

$$(62) \qquad \underline{\mathcal{J}}_n = \frac{\varepsilon_r - 1}{\varepsilon_r} \cos(k_\zeta \zeta) \; (\hat{z} \times \underline{J}_n') \qquad \text{(TE case)}$$

and $\underline{J}_n'$ is the derivative of the expansion functions $\underline{J}_n$ (Eqs. (30) and (31)) employed for the surface-current density $\underline{J}_s$.

If one takes the finite conductivity into account and uses the impedance boundary condition, the magnetic surface-current density can be expanded as

$$(63) \qquad \underline{M}_s = \sum_{n=1}^{N} I_n \underline{M}_n$$

with

$$\underline{M}_n = z_s(\underline{J}_n \times \hat{n}) \qquad \text{(TE case)} \tag{64}$$

Thus, the electromagnetic model, in the TE case, for the problem shown in Fig. 19a is an array of N overlapping, sinusoidal-dipole sources radiating in free space as illustrated in Fig. 19b. Each source is a modal current distribution $\underline{S}_n$, which is comprised of the mode currents $\underline{J}_n$, $\underline{\mathcal{J}}_n$ and $\underline{M}_n$, with weighting I_n.

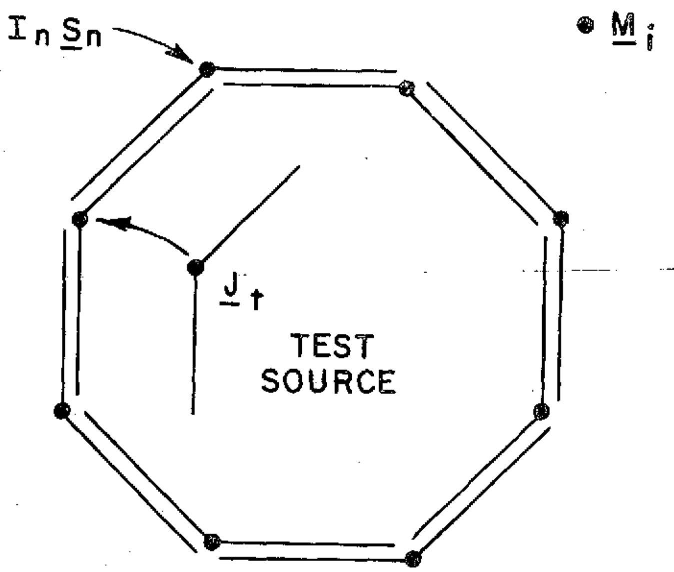

Fig. 19b--Electromagnetic model for the problem of Fig. 19a.

Following the same argument used in the previous section, one performs N independent reaction tests by moving an electric test source to the conducting surface all illustrated in Fig. 19b. Again, the zero-reaction concept leads to the matrix equation, Eq. (9), which can be solved numerically for the stationary current samples.

In the reaction formulation, a polygon cylinder with N segments is employed to represent a cylinder with arbitrary contour. It is found that in order to obtain accurate results one must use at least five segments per wavelength.

C. PLATES AND CORNER REFLECTORS WITH PERFECT CONDUCTIVITY

Electromagnetic modeling of dielectric-coated, conducting bodies with finite extent can be accomplished through the procedure described in the previous sections. In this section, however, two specific cases, namely, scattering from perfectly-conducting plates and corner reflectors are discussed. The plates and corner reflectors are assumed to have an infinitesimal thickness. Therefore, the total surface-current density $\underline{J}_S$ (the vector sum of the current density on the front and back of the plates) is employed in the analysis.

Consider the problems of plane wave scattering by rectangular plates and corner reflectors. The planar surfaces are divided into rectangular cells as illustrated in Fig. 20. The surface-current density is then expanded into two orthogonal sets of overlapping dipole-mode currents. Each dipole-mode current covers two cells and has a sinusoidal current distribution as defined by Eqs. (39) and (40). In Fig. 20 the arrows represent the mode current densities $\underline{J}_m$. Thus, in the reaction calculation, the plates or the corner reflectors are represented by an array of overlapping mode currents radiating in free space and the reaction tests are enforced with a set of electric test sources.

The zero-reaction concept leads again to the matrix equation, Eq. (9), which is then solved for the samples of the surface-current density.

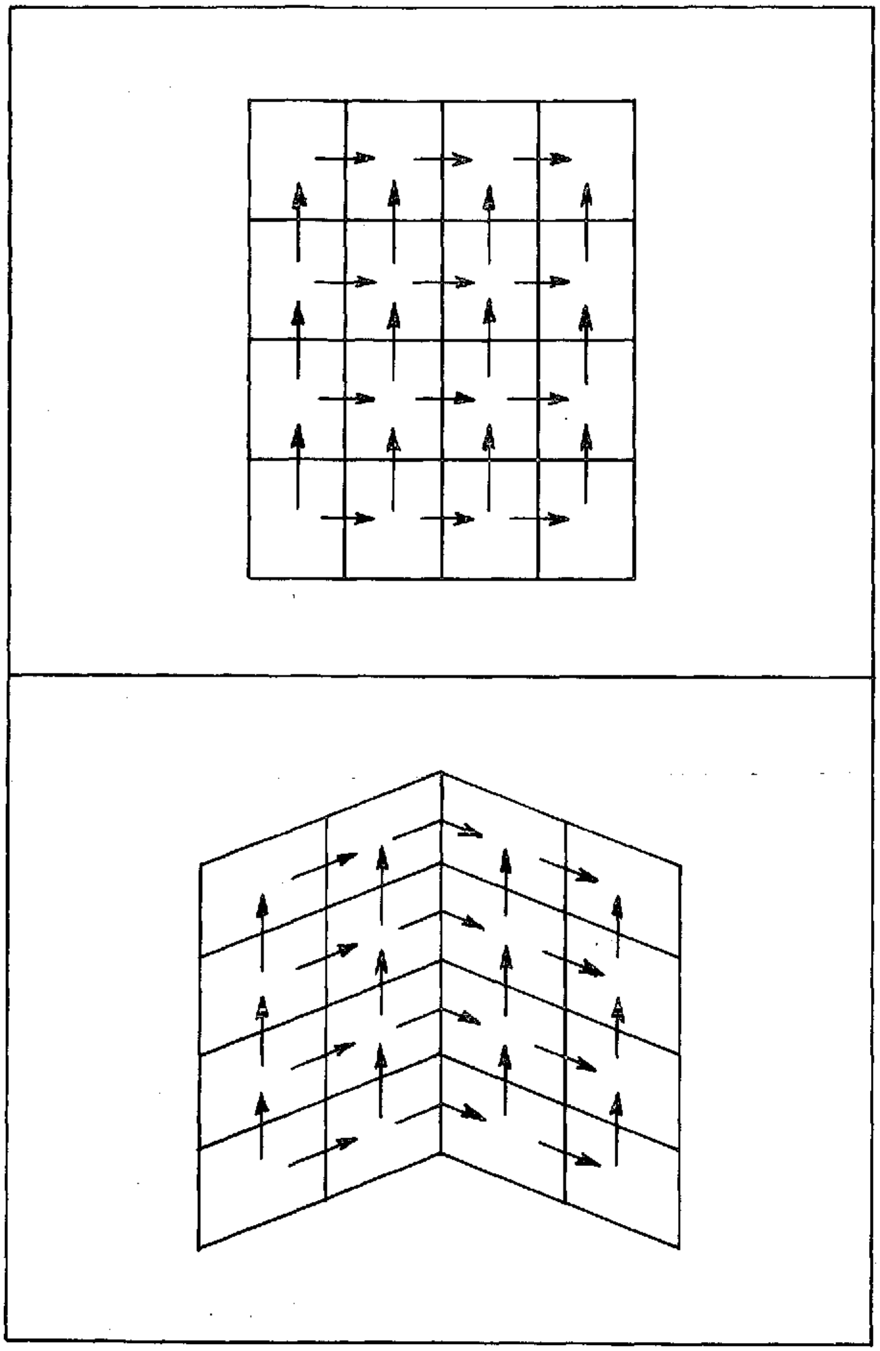

Fig. 20--Electromagnetic model of plate and corner reflector.

CHAPTER VII
FAR-FIELD RADIATION AND SCATTERING

The field scattered by a conducting body is the sum of the free space fields generated by the electric surface-current distribution $\underline{J}_s$ and the magnetic surface-currents $\underline{M}_s$ induced on the conducting surface. If the body has a dielectric coating, the contribution from the polarization-current density $\underline{J}_{eq}$ should be included. To obtain the total field one adds the free space field of the impressed current source $(\underline{J}_i, \underline{M}_i)$.

In the reaction calculation the continuous conducting surface is segmentized and the current distribution is represented by a set of modal currents. Thus, the field scattered by a conducting body is the sum of the free space fields generated by these modal currents. The far-field contribution due to three types of modal currents employed in this dissertation will be examined in the following sections.

A. LONGITUDINAL, DIELECTRIC-COATED STRIP SOURCE

For an electric line source J_z of infinite length located on the z axis, the free space field is

(65) $$\underline{E} = -\hat{z}k\eta \; J_z \; H_o^{(2)}(k\rho)/4$$

(66) $$\underline{H} = -\hat{\phi}j \; J_z \; H_1^{(2)}(k\rho)/4k .$$

If the electric line source is parallel with the z axis and passes through the point (x,y), its free space field at a distant point (ρ, ϕ) is

(67) $$\underline{E} = -\hat{z} \frac{k\eta \; J_z\sqrt{2j} \; e^{-jk\rho}}{4\sqrt{\pi k\rho}} e^{jk(x \cos\phi - y \sin\phi)} .$$

Fig. 21 illustrates an electric surface-current distribution $\underline{J}(t) = \hat{z} \; J(t)$ on a dielectric-coated, conducting strip extending from (x_1,y_1) to (x_2,y_2). Distance along the strip is measured by the coordinate t, and distance perpendicular to the strip is measured by the coordinate ζ. For an arbitrary point on the strip,

(68) $$x = x_1 + t \cos\alpha$$

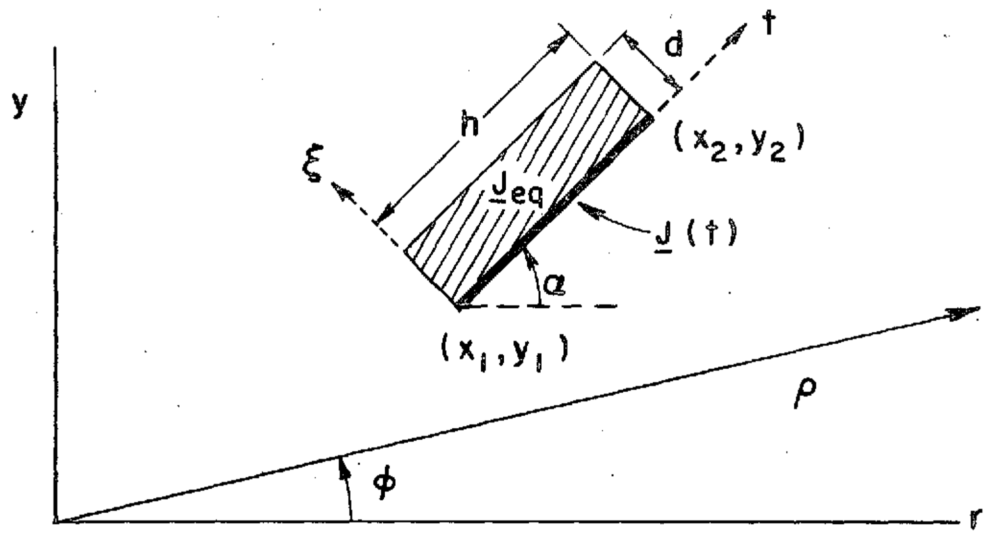

Fig. 21--A dielectric-coated strip source.

(69) $\quad y = y_1 + t \sin \alpha$.

From Eq. (67), the free space field of this source $\underline{J}(t) = \hat{z}\, J_z(t)$ at a distant point (ρ, ϕ) is

(70) $$\underline{E}^J = -\hat{z}\, \frac{k\eta\sqrt{2j}}{4\sqrt{\pi k\rho}}\; e^{-jk\rho}\; e^{j\psi_1} \int_0^h J_z(t)\, e^{jkct}\, dt$$

where

(71) $$\psi_1 = k(x_1 \cos\phi + y_1 \sin\phi)$$

(72) $$c = \cos(\alpha - \phi)$$

Using the approximation (Eq. (55)), the far-field contribution from the volume current density $\underline{J}_{eq}$ is

$$(73) \qquad \underline{E}^{J_{eq}} = \hat{z}\,\frac{k\eta\sqrt{2j}\,e^{-jk\rho}}{4\sqrt{\pi k\rho}}\,e^{j\psi_1}\left(\frac{\varepsilon_r-1}{\varepsilon_r}\right)$$

$$\int_0^h \int_0^d k_1 \sin(k_1\zeta)\; J_z(t)e^{jkct}\, e^{-jk\zeta\sin(\alpha-\phi)}\, d\zeta\, dt\,.$$

The integration along ζ can be readily performed to yield

$$(74) \qquad \underline{E}^{J_{eq}} = g_{TM}\,\underline{E}^{J}$$

where

$$(75) \qquad g_{TM} = \left(\frac{\varepsilon_r-1}{2\varepsilon_r}\right)k_1\left[\frac{e^{j[k_1-k\sin(\alpha-\phi)]d}-1}{k_1-k\sin(\alpha-\phi)} + \frac{e^{-j[k_1+k\sin(\alpha-\phi)]d}-1}{k_1+k\sin(\alpha-\phi)}\right]$$

If one introduces finite conductivity to the conducting strip and uses the impedance boundary condition (Eq. (6)), the free space field due to this magnetic current distribution is

$$(76) \qquad \underline{E}^{M} = \frac{Z_s\sin(\alpha-\phi)}{\eta}\,\underline{E}^{J}\,.$$

From Eqs. (70), (74), and (76), the distant scattered field generated by the dielectric-coated conducting strip (shown in Fig. 21) with $J_z(t) = 1$ is

$$(77) \qquad E_z^s = \frac{-\eta\, e^{-j\pi/4}}{\sqrt{8\pi k\rho}}\, e^{-jk\rho}\left(1 + g_{TM} + \frac{Z_s\sin(\alpha-\phi)}{\eta}\right)F_{TM}$$

where

$$(78) \qquad F_{TM} = \frac{e^{j\psi_2} - e^{j\psi_1}}{c}$$

$$(79) \qquad \psi_2 = k(x_2\cos\phi + y_2\sin\phi)\,.$$

For broadside direction, i.e., $\alpha - \phi = \pi/2$ or $3\pi/2$,

(80) $$F_{TM} = jkh\, e^{j\psi_1} .$$

B. TRANSVERSE, DIELECTRIC-COATED STRIP MONOPOLE

Consider again the dielectric-coated strip monopole shown in Fig. 21. This time the source is a transverse electric surface-current distribution $\underline{J}(t) = \hat{t}\, J_t(t)$. From Eq. (24), the magnetic field at a distant point (ρ, ϕ) is

(81) $$\underline{H}^J = -\hat{z}\, \frac{k\sqrt{2j}\, \sin(\alpha-\phi)}{4\sqrt{\pi k\rho}}\, e^{-jk\rho}\, e^{j\psi_1} \int_0^h J_t(t)\, e^{jkct}\, dt$$

From Eq. (62), the far-field contribution from the volume current density is

(82) $$\underline{H}^{J_{eq}} = \hat{z}\, \frac{k\sqrt{2j}\, \cos(\alpha-\phi)}{4\sqrt{\pi k\rho}}\, e^{-jk\rho}\, e^{j\psi_1} \left(\frac{\varepsilon_r - 1}{\varepsilon_r}\right)$$

$$\int_0^h \int_0^d \cos(k_\zeta \zeta)\, J_t'(t)\, e^{jkct}\, e^{-jk\sin(\alpha-\phi)\zeta}\, d\zeta\, dt .$$

The integration along ζ is performed to yield

(83) $$\underline{H}^{J_{eq}} = \frac{\hat{z}k\sqrt{2j}}{4\sqrt{\pi k\rho}} \cos(\alpha-\phi)\, e^{-jk\rho}\, e^{j\psi_1}\, g_{TE} \int_0^h J_t'(t)\, e^{jkct}\, dt$$

where

(84) $$g_{TE} = \left(\frac{\varepsilon_r - 1}{2j\varepsilon_r}\right)\left[\frac{e^{j[k_\zeta - k\sin(\alpha-\phi)]d} - 1}{k_\zeta - k\sin(\alpha-\phi)} - \frac{e^{-j[k_\zeta + k\sin(\alpha-\phi)]d} - 1}{k_\zeta + k\sin(\alpha-\phi)}\right]$$

Consider a magnetic surface-current distribution $M_z(t)$ on the planar strip extending from (x_1, y_1) to (x_2, y_2) as in Fig. 21. From duality relations in electric and magnetic systems and Eq. (70), the free space field of this source at a distant point (ρ, ϕ) is

(85) $$\underline{H}^M = \frac{-\hat{z}k\sqrt{2j}\, e^{-jk\rho}}{4\eta\sqrt{\pi k\rho}}\, e^{j\psi_1} \int_0^h M_z(t)\, e^{jkct}\, dt .$$

If the magnetic surface-current density $M_z(t)$ in Eq. (85) arises through the finite conductivity of the cylinder, the impedance boundary condition in Eq. (6) yields

$$M_z(t) = s\, Z_s\, J_t(t) \tag{86}$$

where

$$s = (\hat{t} \times \hat{n}) \cdot \hat{z} = \pm 1 \tag{87}$$

and the unit normal $\hat{n}$ is directed into the source region.

From Eqs. (81), (83), (85), and (86), the distant scattered field from one segment of the dielectric-coated conducting cylinder (the segment in Fig. 21) is

$$H_z^s = - \frac{\sqrt{2j}\left[(\eta \sin(\alpha-\phi) + s\, Z_s) F_1 - \eta \cos(\alpha-\phi)\, g_{TE}\, F_2\right] e^{-jk\rho}}{4\eta \sin kh \sqrt{\pi k \rho}} \tag{88}$$

where

$$F_1 = k \sin(kh)\, e^{j\psi_1} \int_0^h J_t(t)\, e^{jkct}\, dt \tag{89}$$

and

$$F_2 = k \sin(kh)\, e^{j\psi_1} \int_0^h J_t'(t)\, e^{jkct}\, dt\ . \tag{90}$$

If the transverse electric current on this segment has a sinusoidal distribution as follows

$$J_t(t) = \frac{I_1 \sin(kh-kt) + I_2 \sin(kt)}{\sin(kh)}\ , \tag{91}$$

Eqs. (89) and (90) yield

$$(92)\qquad F_1 = \frac{I_1}{\sin^2(\alpha-\phi)}\left[e^{j\psi_2} -(\cos kh + j\,c\sin kh)\,e^{j\psi_1}\right] + \frac{I_2}{\sin^2(\alpha-\phi)}\left[e^{j\psi_1} -(\cos kh - j\,c\sin kh)\,e^{j\psi_2}\right]$$

and

$$(93)\qquad F_2 = \frac{I_1}{\sin^2(\alpha-\phi)}\left[j\,c\,e^{j\psi_2} + (\sin kh - j\,c\cos kh)\,e^{j\psi_1}\right] + \frac{I_2}{\sin^2(\alpha-\phi)}\left[j\,c\,e^{j\psi_1} - (\sin kh + j\,c\cos kh)\,e^{j\psi_2}\right].$$

In the end-fire direction where $(\alpha-\phi)$ is zero or π,

$$(94)\qquad F_1 = \left[e^{j\psi_1}\sin kh - kh\,e^{j\psi_2}\right] j\,c\,I_1/2 - \left[e^{j\psi_2}\sin kh - kh\,e^{j\psi_1}\right] j\,c\,I_2/2$$

and

$$(95)\qquad F_2 = \left[kh\,e^{j\psi_2} + \sin kh\,e^{j\psi_1}\right] I_1/2 - \left[kh\,e^{j\psi_1} + \sin kh\,e^{j\psi_2}\right] I_2/2 \quad .$$

C. RECTANGULAR SURFACE DIPOLE

Consider an electric surface dipole with current density $\underline{J} = \hat{z}'\cos(\pi z'/a)$ located on the y'z' plane as shown in Fig. 22. From reciprocity, the free space electric field generated by this source at a distant point (r, θ_s', ϕ_s') (from Eq. (53)) is

$$(96)\qquad \underline{E}^s = \hat{\theta}_s' \frac{j\omega\mu}{2}\, a\, b\, \sin\theta_s' \frac{\sin(X_s)}{X_s}\,\frac{\cos(Y_s/2)}{(Y_s^{\,2}-\pi^2)}\,\frac{e^{-jkr}}{r}$$

where

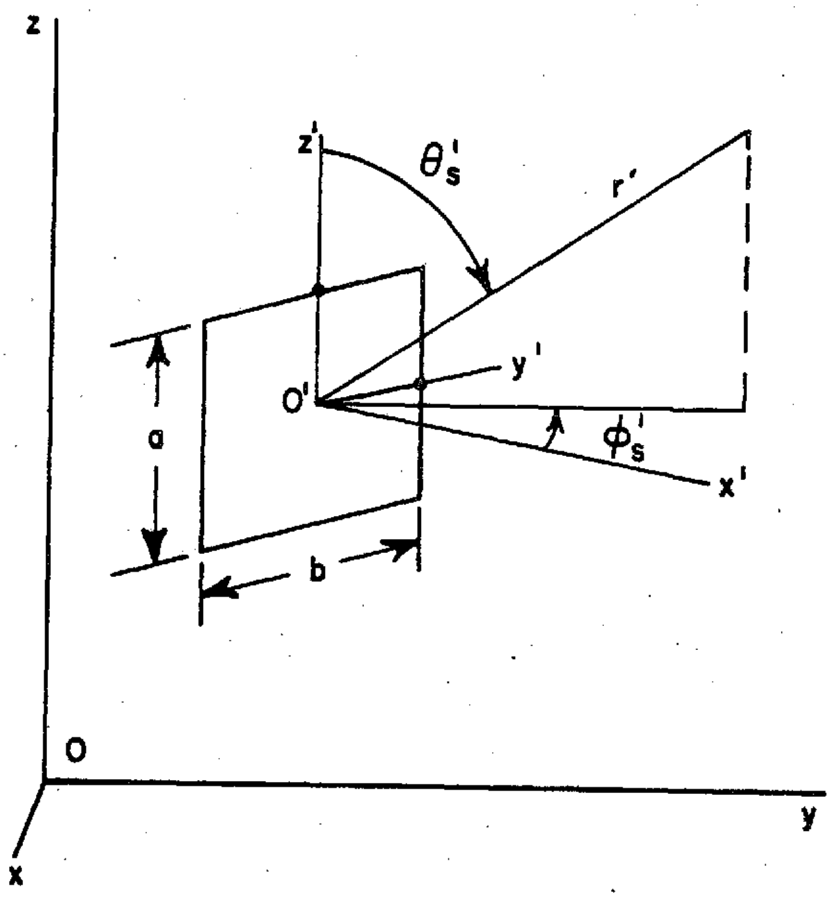

Fig. 22--A surface dipole radiates in free space.

(97) $X_s = 0.5\ kb\ \sin\theta_s' \sin\phi_s'$

(98) $Y_s = ka\ \cos\theta_s'$.

The $\hat{\theta}$- and $\hat{\phi}$-components of the scattered field with respect to the reference coordinate system 0 can be obtained easily via appropriate coordinate transformation.

In plane-wave scattering problems, one is usually interested in the echo information defined as follows:

$$(99) \qquad \text{echo width} = \lim_{\rho\to\infty} 2\pi\rho \frac{|\underline{E}^s|^2}{|\underline{E}^i|^2} \qquad \text{(TM case)} ,$$

$$(100) \qquad \text{echo width} = \lim_{\rho\to\infty} 2\pi\rho \frac{|\underline{H}^s|^2}{|\underline{H}^i|^2} \qquad \text{(TE case)} ,$$

and

$$(101) \qquad \text{echo area} = \lim_{r\to\infty} 4\pi r^2 \frac{|\underline{E}^s|^2}{|\underline{E}^i|^2} \qquad \text{(three-dimensional case)}.$$

In three-dimensional antenna problems, one is interested in the power gain:

$$(102) \qquad \text{gain} = \frac{4\pi r^2 \; |\underline{E}|^2}{\eta |V|^2 G}$$

where V is the terminal voltage and G is the conductance of the antenna.

CHAPTER VIII
NUMERICAL RESULTS

A. TM DIELECTRIC-COATED CYLINDERS

In this section numerical results are presented for the backscattering echo width of cylinders with thin dielectric coating. The incident wave is a transverse-magnetic plane wave. Fig. 23 presents the backscattering echo width of a circular cylinder with a thin dielectric coating. In the reaction calculation the cylinder is divided into N segments with $N = 12 + 20\ d/\lambda$ and d is the diameter. Similar results for a square cylinder with 16 segments are shown in Fig. 24. For comparison, similar results are presented in Figs. 25 and 26 for uncoated cylinders with finite conductivity. Bistatic echo width as a function of aspect angle is presented in Fig. 27 through 29 for circular and square cylinders with a thin dielectric coating. In all cases, the dielectric layer has a relative dielectric constant of 10.

B. TE DIELECTRIC-COATED CYLINDERS

Numerical results are presented for cylinders with a thin dielectric coating illuminated by a transverse-electric incident plane wave. Fig. 30 presents the backscattering echo width of a circular cylinder with a thin dielectric coating. As before, the cylinder is divided into N segments with $N = 12 + 20\ d/\lambda$ and d is the diameter. Similar results for a square cylinder with 16 segments are presented in Fig. 31. In Figs. 30 and 31, backscattering echo width for uncoated cylinders are also included for comparison. Bistatic echo width as a function of observation angle for coated cylinders are shown in Figs. 32, 33 and 34. The relative dielectric constant is 1.5 in each case.

Examining the results, one can note that the reaction calculation gives accurate data for uncoated conducting cylinders. Satisfactory results are also obtained for cylinders with a thin dielectric coating.

C. PERFECTLY-CONDUCTING PLATES AND CORNER REFLECTORS

Fig. 35 presents the backscattering echo area of a square plate with perfect conductivity for the broadside aspect. In the reaction calculation, the plate is divided into cells, and overlapping current modes were employed as illustrated in Fig. 36. In this case the transverse current was neglected and 45 modes were used for the current distribution. Useful results can be obtained with as few as one mode per square wavelength of surface area. For comparison, Fig. 35 also shows the experimental measurements of Kouyoumjian [18].

The magnitude and phase of the induced current density on a perfectly-conducting rectangular plate are illustrated in Figs. 37 and

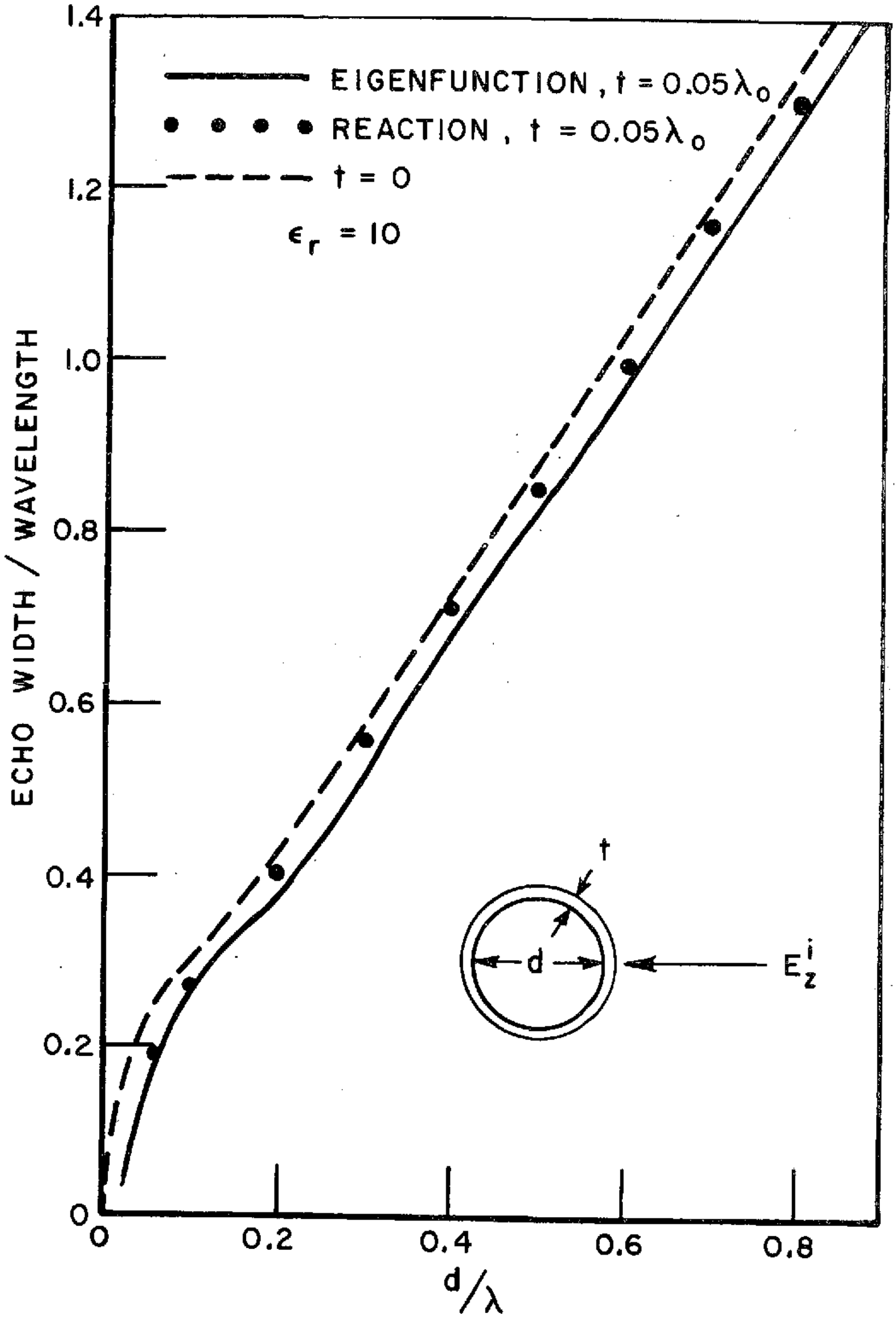

Fig. 23--Backscattering echo width of dielectric-coated circular cylinder.

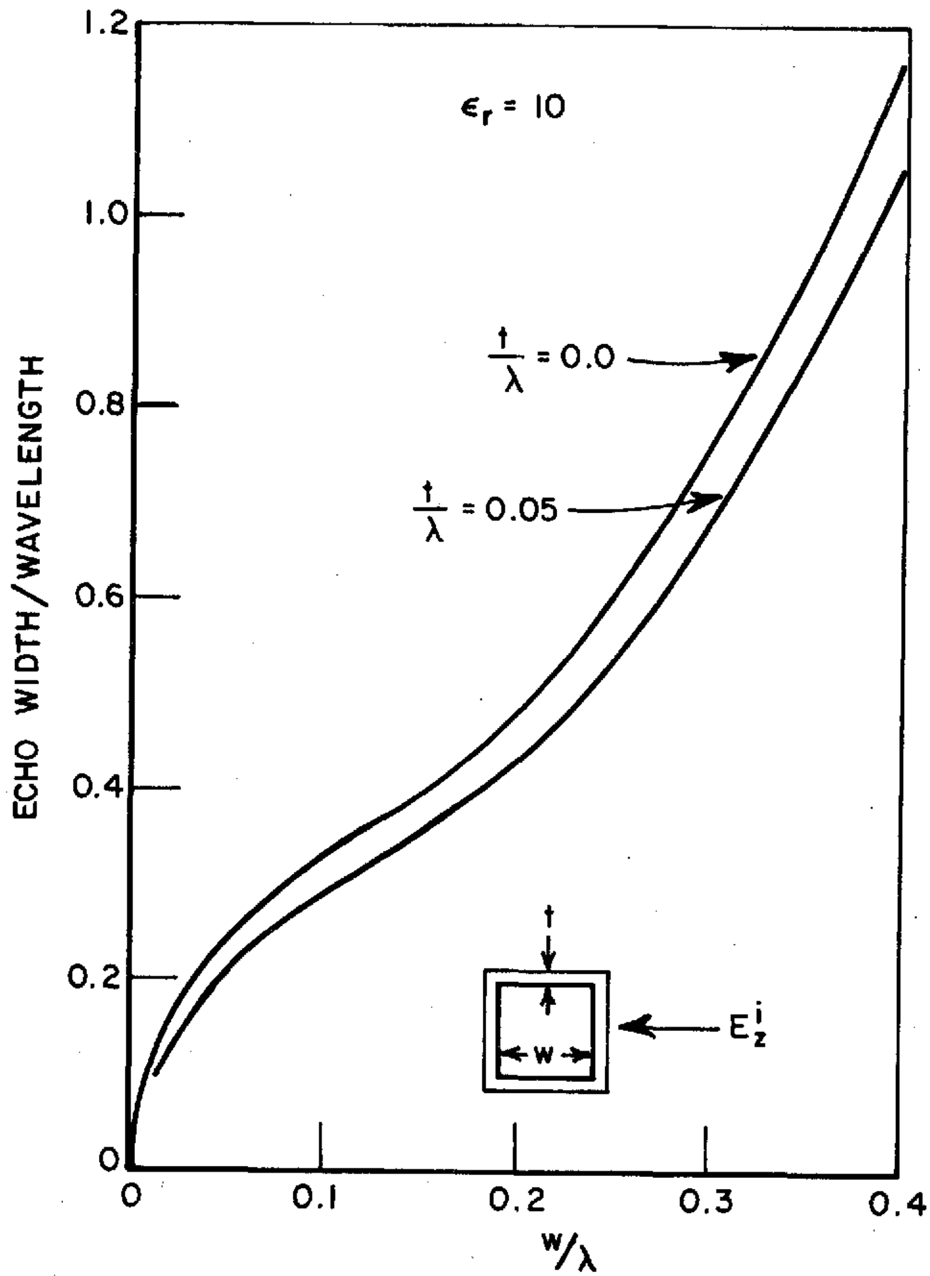

Fig. 24--Broadside backscattering echo width of dielectric-coated square cylinder.

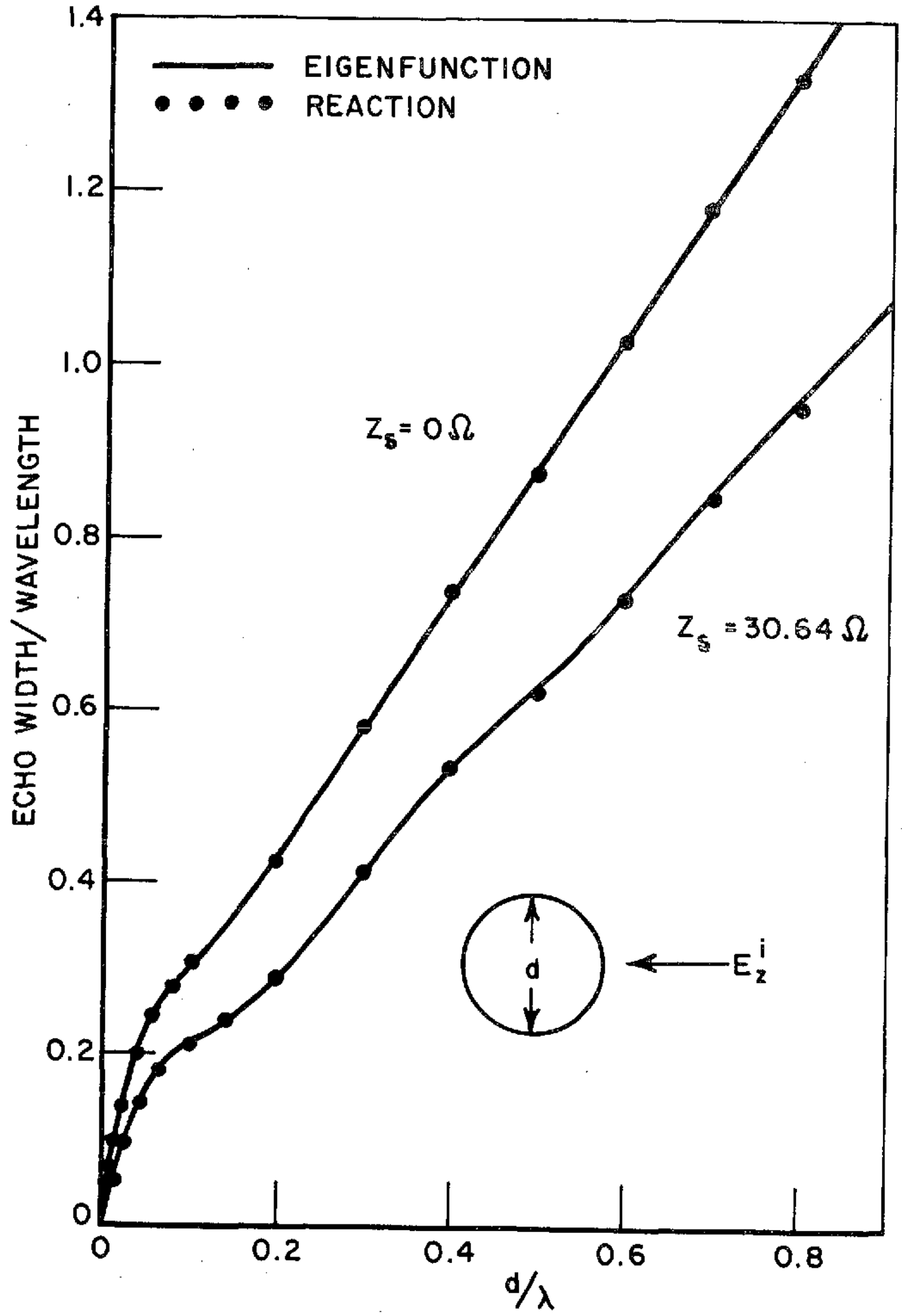

Fig. 25--Backscattering echo width of circular cylinder.

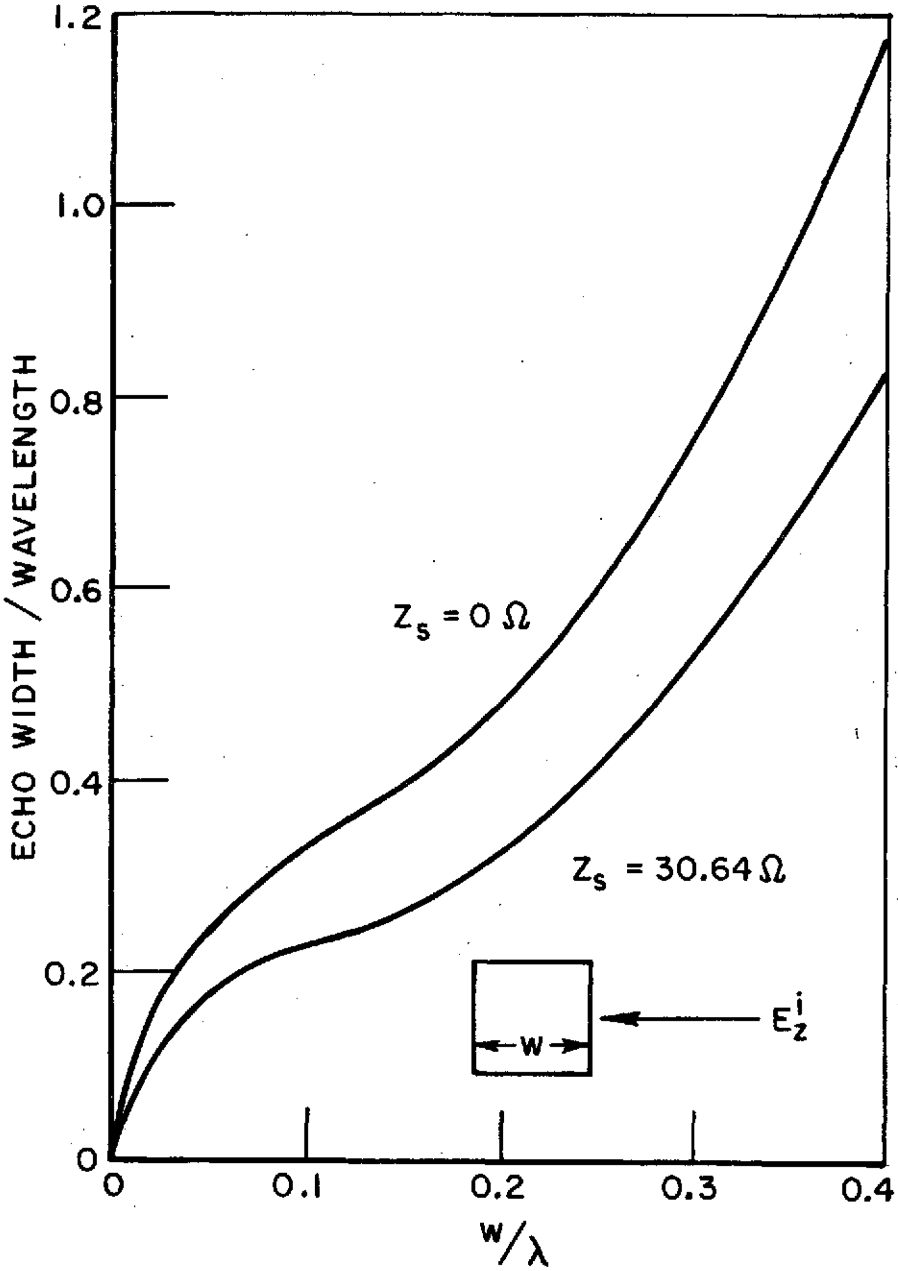

Fig. 26--Broadside backscattering echo width of square cylinder.

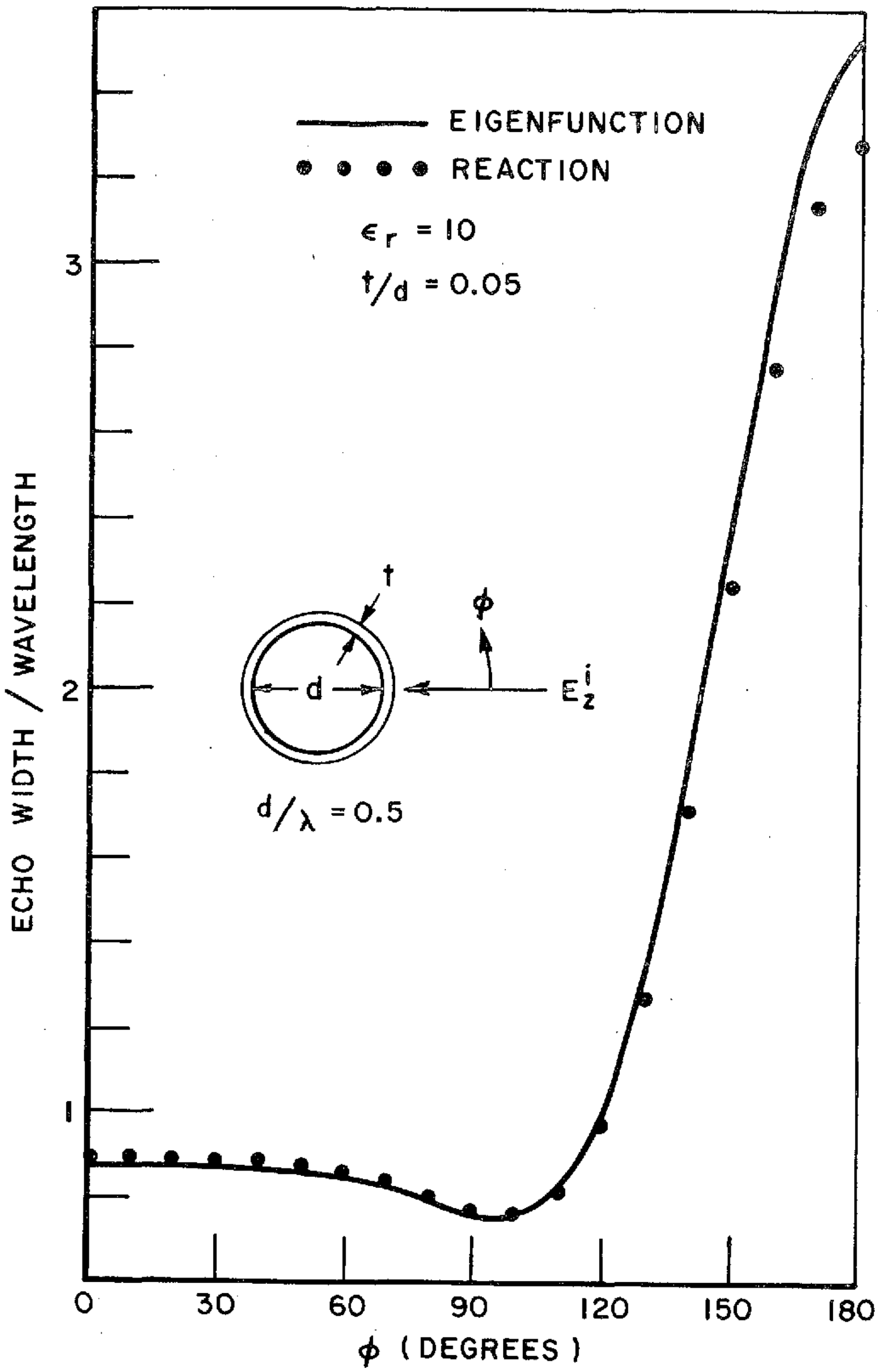

Fig. 27--Bistatic echo width of dielectric-coated circular cylinder.

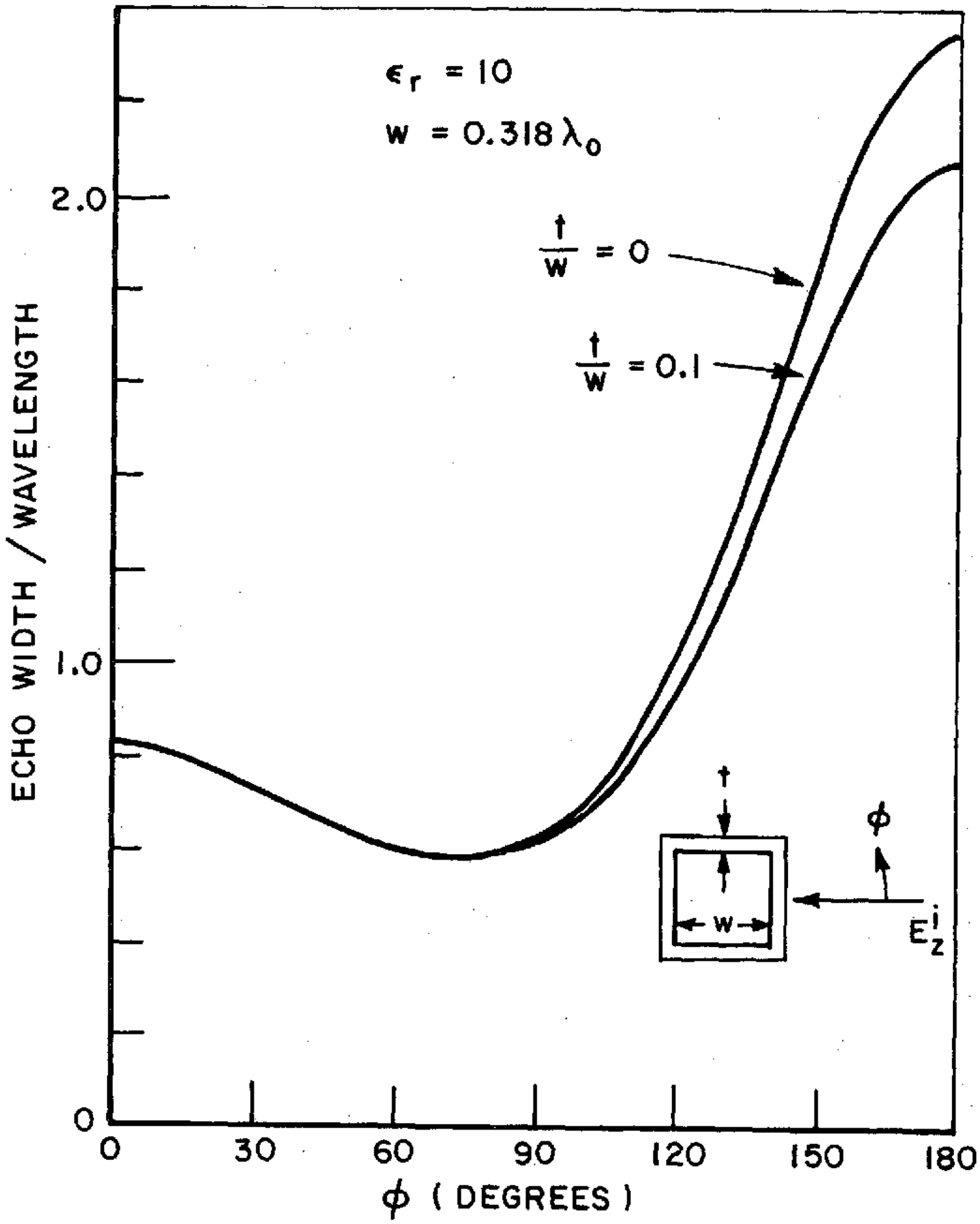

Fig. 28--Bistatic echo width of dielectric-coated square cylinder for broadside incidence.

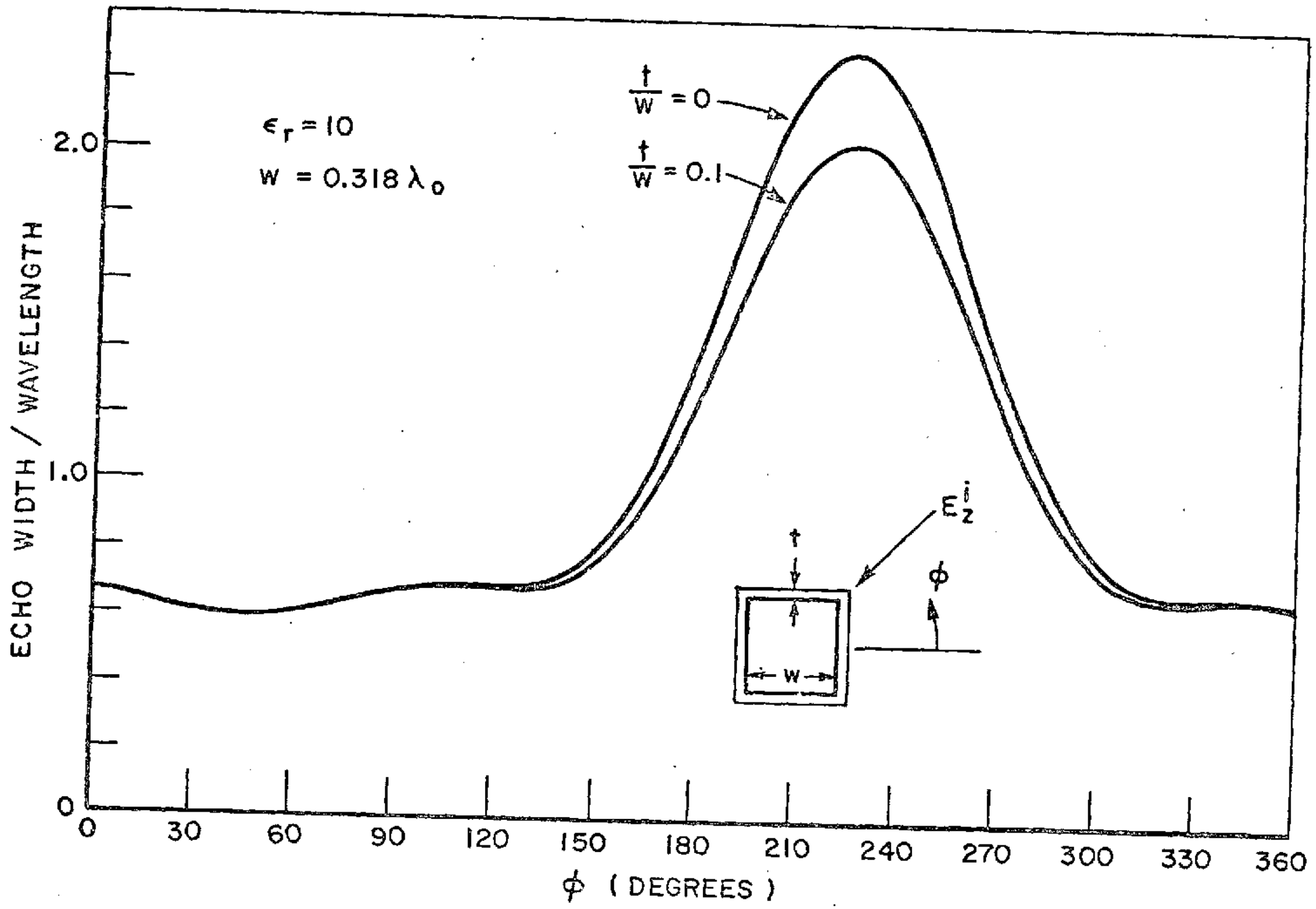

Fig. 29--Bistatic echo width of dielectric-coated square cylinder for oblique incidence.

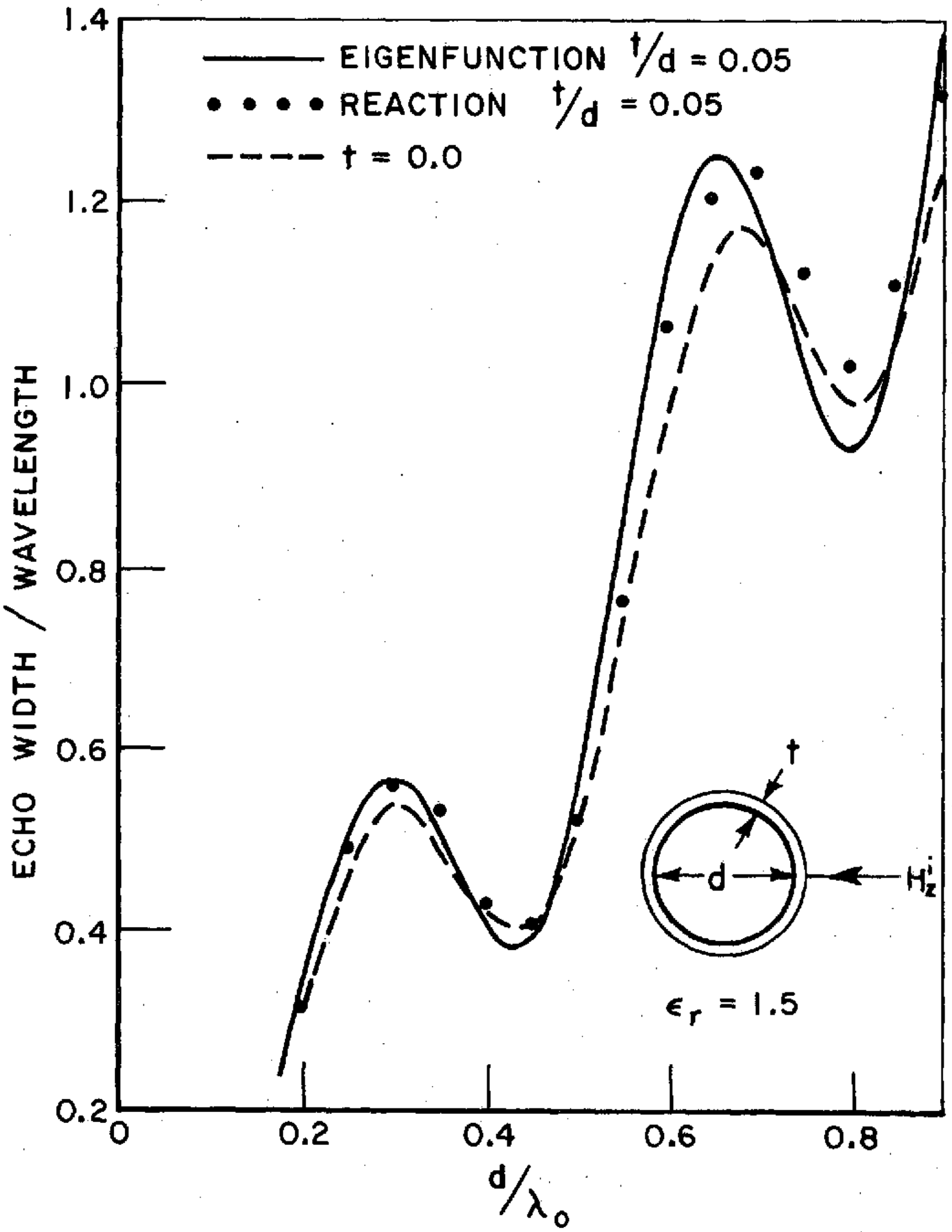

Fig. 30--Backscattering echo width of dielectric-coated circular cylinder.

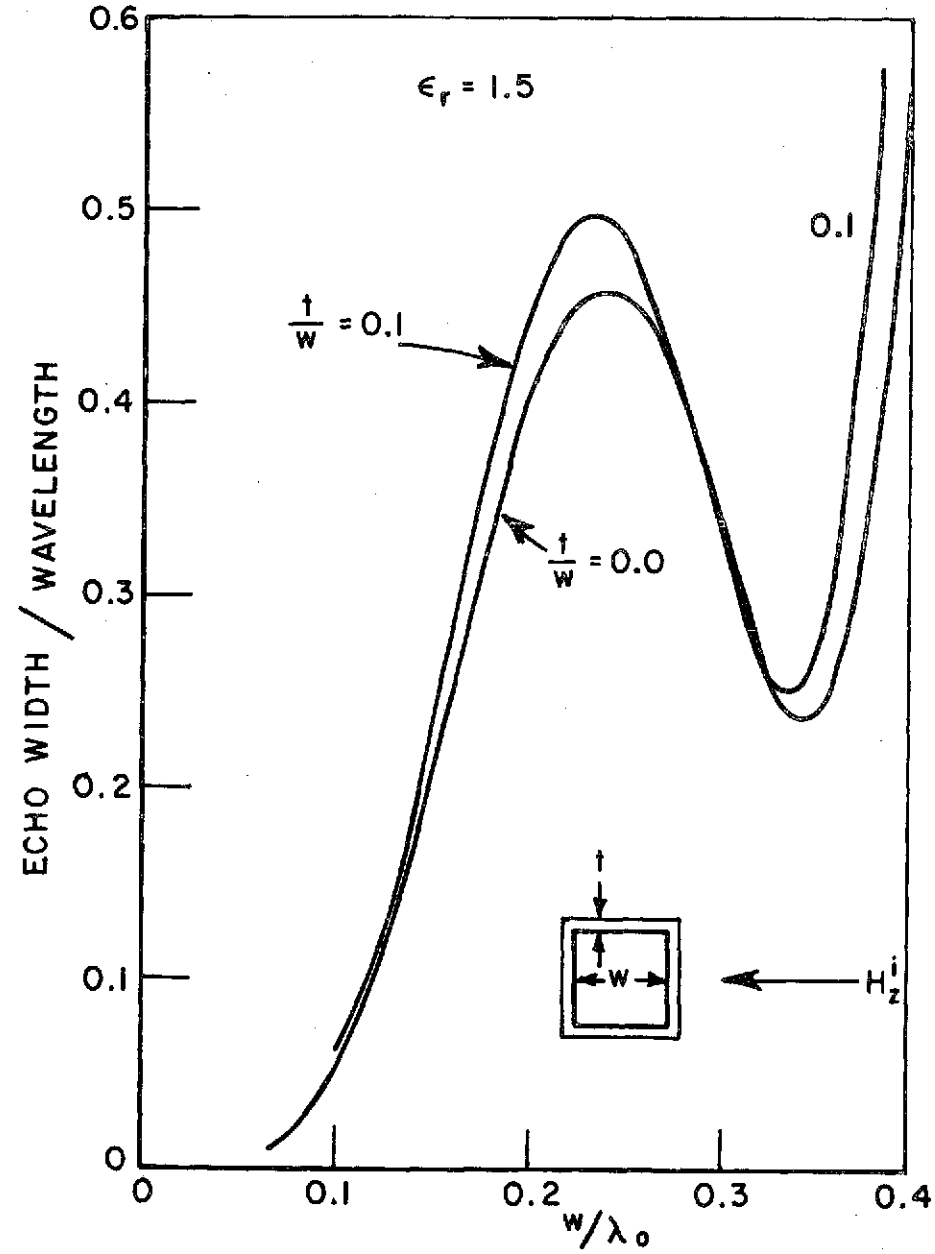

Fig. 31--Broadside backscattering echo width of dielectric-coated square cylinder.

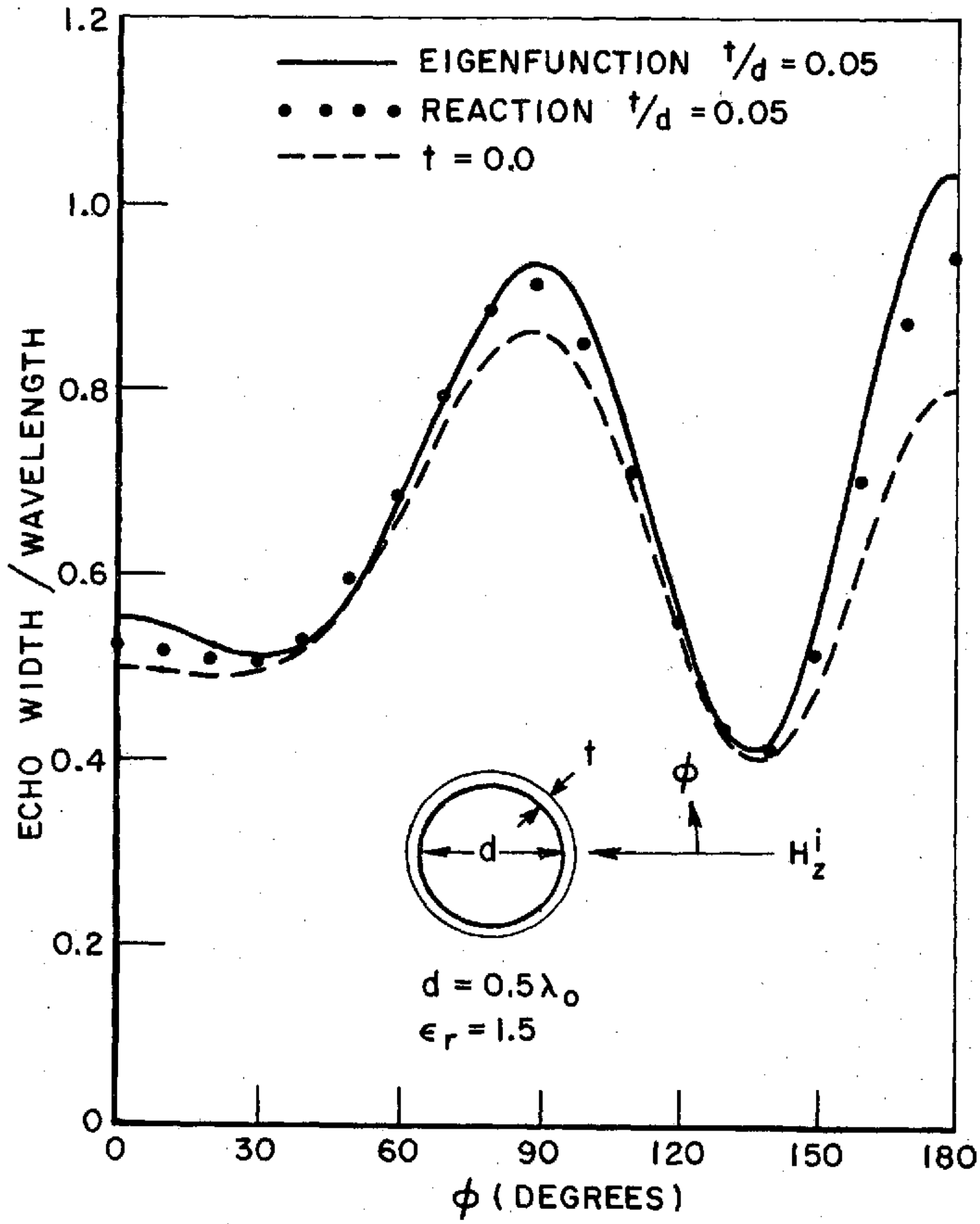

Fig. 32--Bistatic echo width of dielectric-coated circular cylinder.

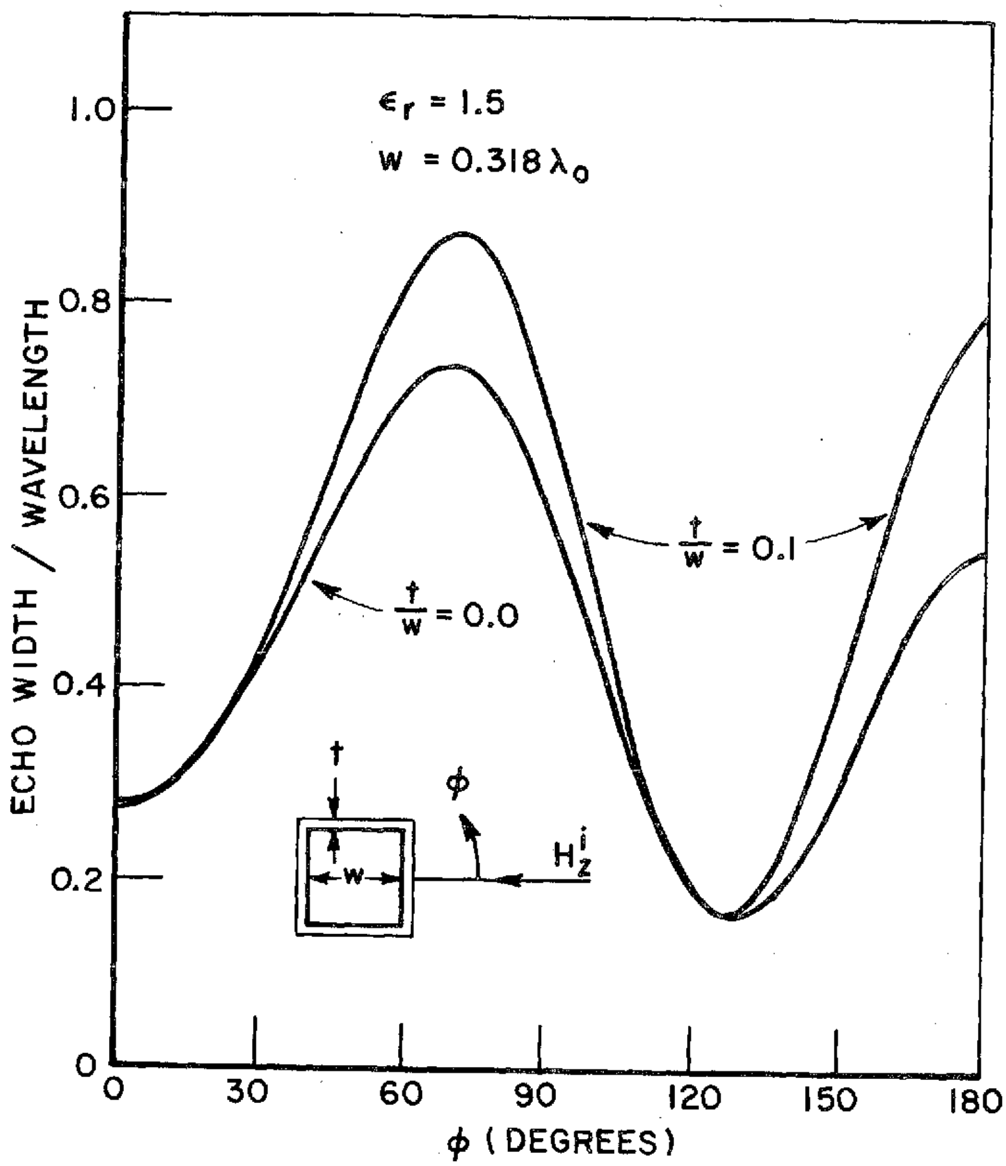

Fig. 33--Bistatic echo width of dielectric-coated square cylinder for broadside incidence.

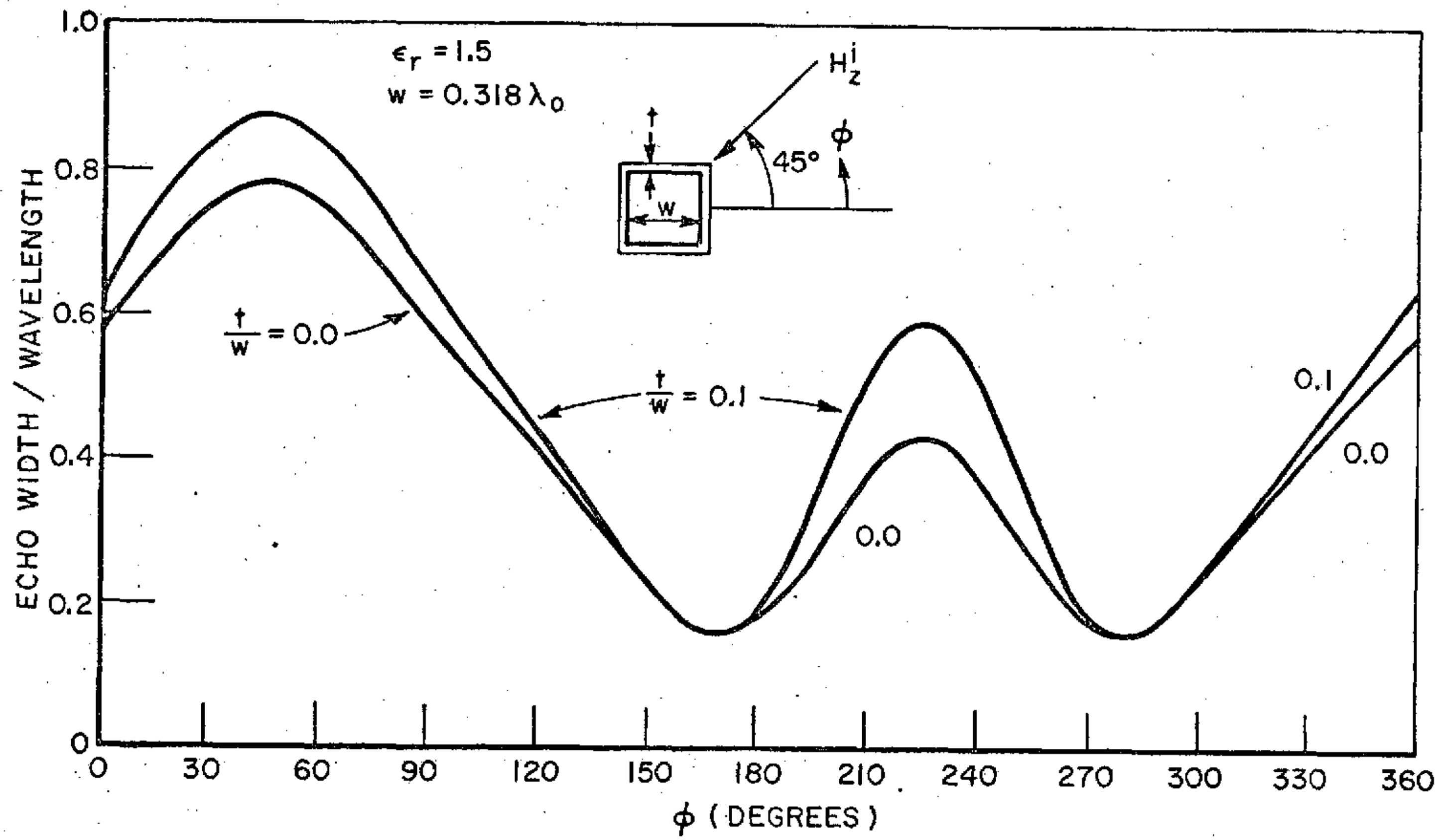

Fig. 34--Bistatic echo width of dielectric-coated square cylinder for oblique incidence.

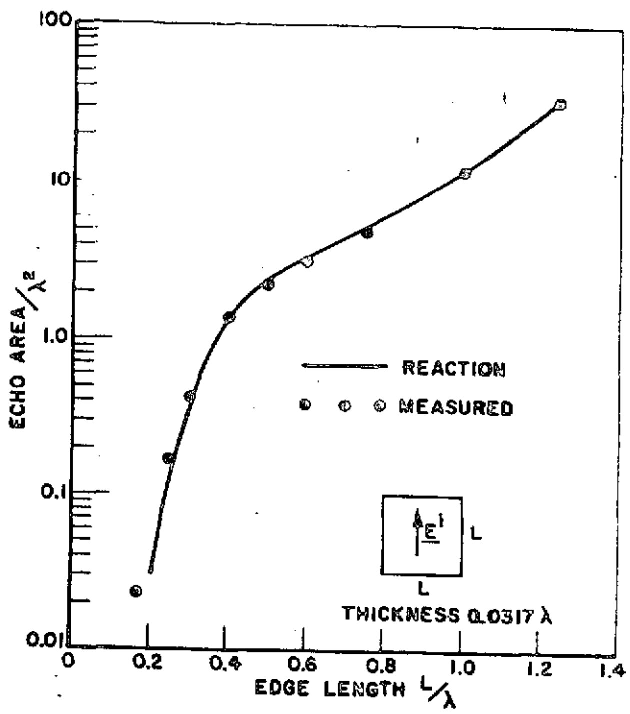

Fig. 35--Backscatter cross-section of perfectly conducting square plate for the broadside aspect.

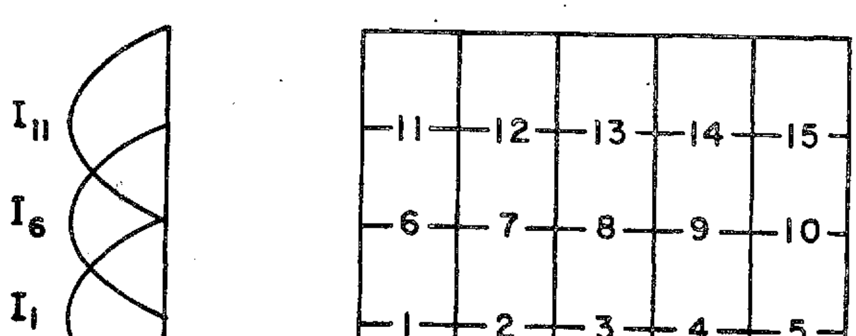

Fig. 36--Electromagnetic modeling of plate.

38. Figs. 39 through 42 show the normalized backscatter cross section of a rectangular plate. Figs. 43 and 44 show the normalized backscatter cross section of a corner reflector. The title of each figure gives the echo area at the broadside aspect in terms of dB = 10 log(σ/λ^2).

Figs. 46 through 49 show the E-plane gain of the corner-reflector antenna illustrated in Fig. 45. Figs. 50 through 53 show the H-plane gain of the same antenna. For comparison, Figs. 46 through 53 include experimental measurements obtained by Melvin Gilreath at NASA Langley Research Center. In the experimental measurements the receiving antenna was linearly polarized in the theta direction. Similarly, the calculated gain is based on E_θ. The dipole length is $\lambda/2$ and the radius is 0.005λ.

In the reaction calculation, only vertical modes were employed to approximate the current distribution. The number of modes used to obtain the results given in Figs. 37 through 53 are listed below. In each case, the matrix size is equal to the number of modes.

Figs.	Number of Modes
37,38	45
39,40	55
41,42	75
43,44	30
46-53	61

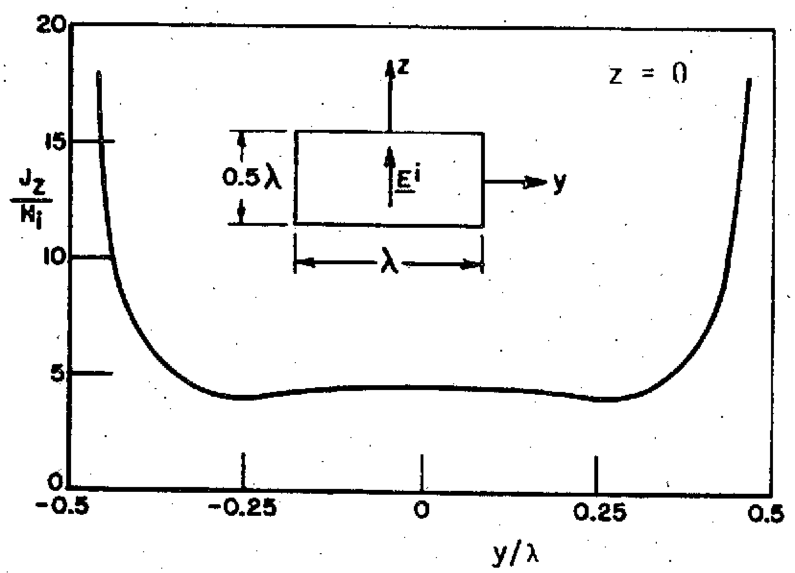

Fig. 37--Magnitude of surface-current density induced on a perfectly-conducting rectangular plate for a plane wave incident at broadside.

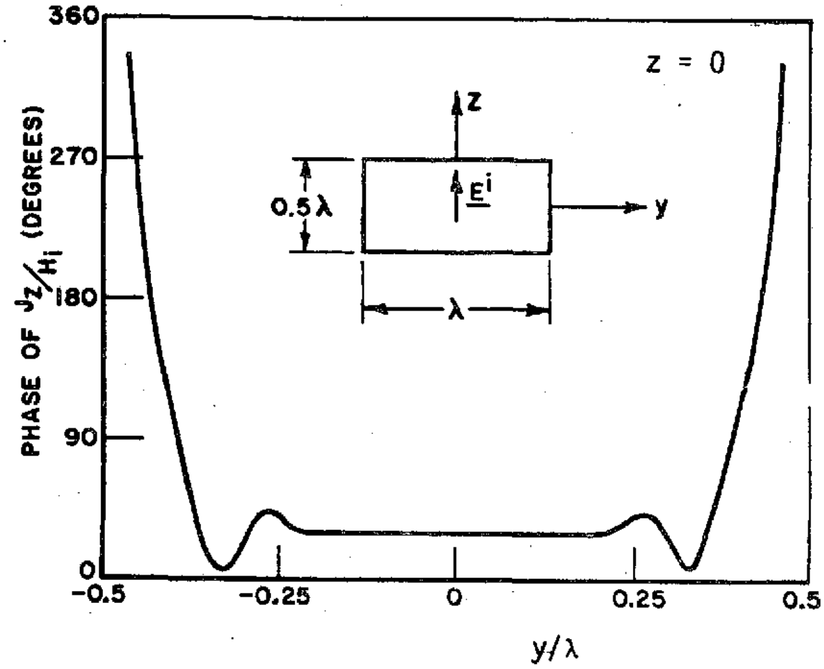

Fig. 38--Phase of surface-current density induced on a rectangular plate for a plane wave incident at broadside.

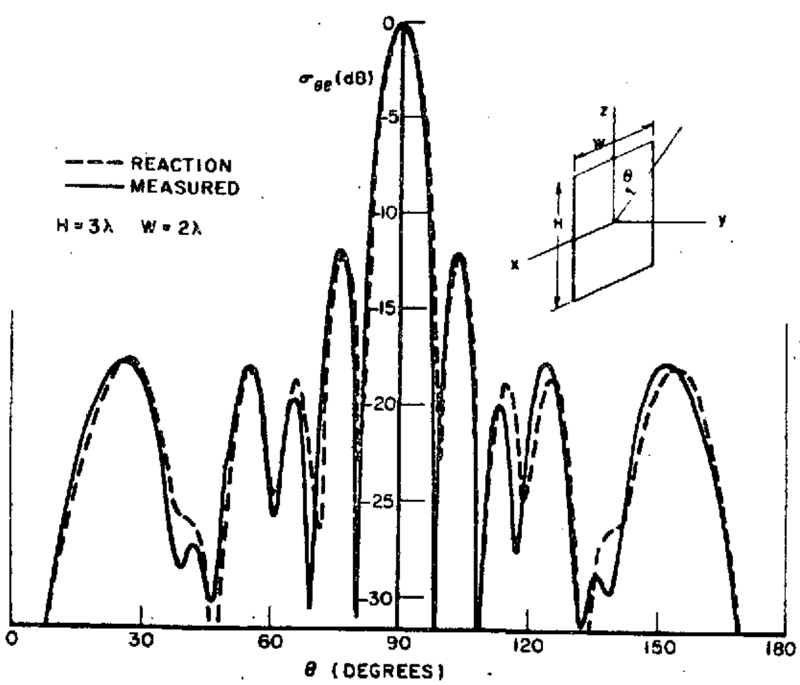

Fig. 39--Normalized backscatter cross-section in the yz plane of a rectangular plate. $\sigma_{\theta\theta}(\theta, \phi)$ = 15.25 dB at (90°,90°).

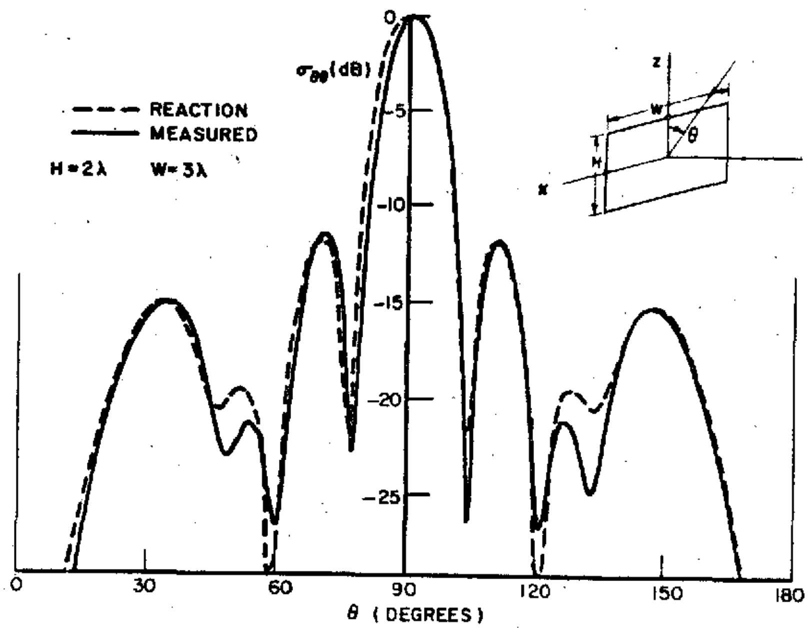

Fig. 40--Normalized backscatter cross-section in the yz plane of a rectangular plate. $\sigma_{\theta\theta}(\theta, \phi)$ = 15.23 dB at (90°,90°).

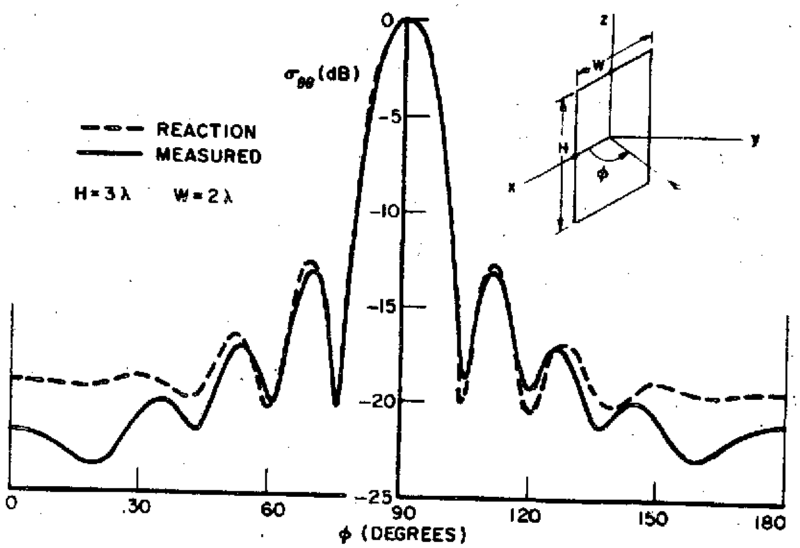

Fig. 41--Normalized backscatter cross-section in the xy plane of a rectangular plate. $\sigma_{\theta\theta}(\theta, \phi)$ = 15.25 dB at (90°,90°).

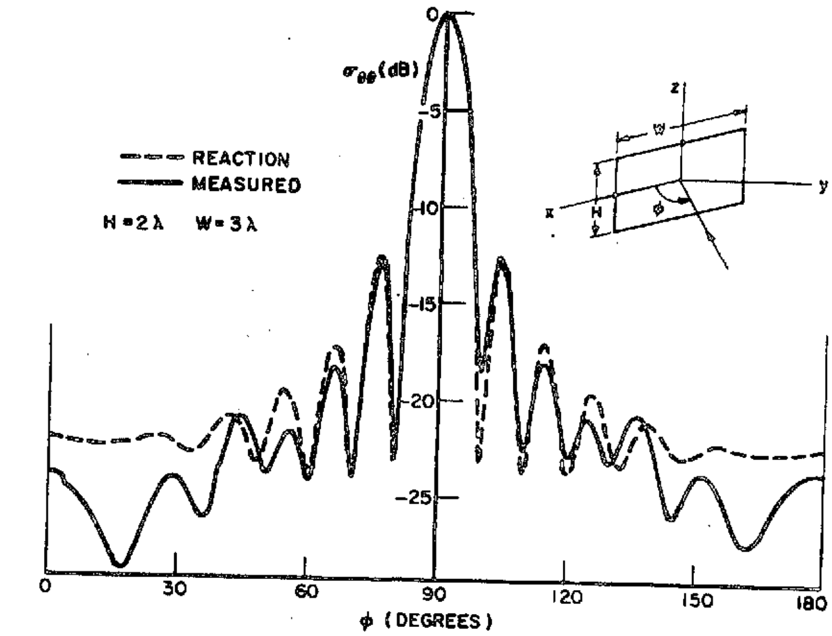

Fig. 42--Normalized backscatter cross-section in the xy plane of a rectangular plate. $\sigma_{\theta\theta}(\theta, \phi)$ = 15.23 dB at (90°,90°).

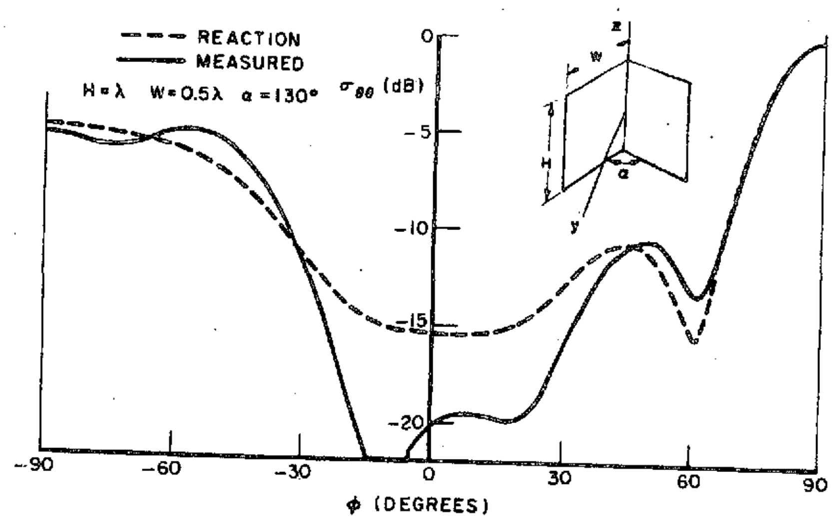

Fig. 43--Normalized backscatter cross-section in the xy plane of a corner reflector. $\sigma_{\theta\theta}(\theta, \phi)$ = -0.38 dB at (90°,90°).

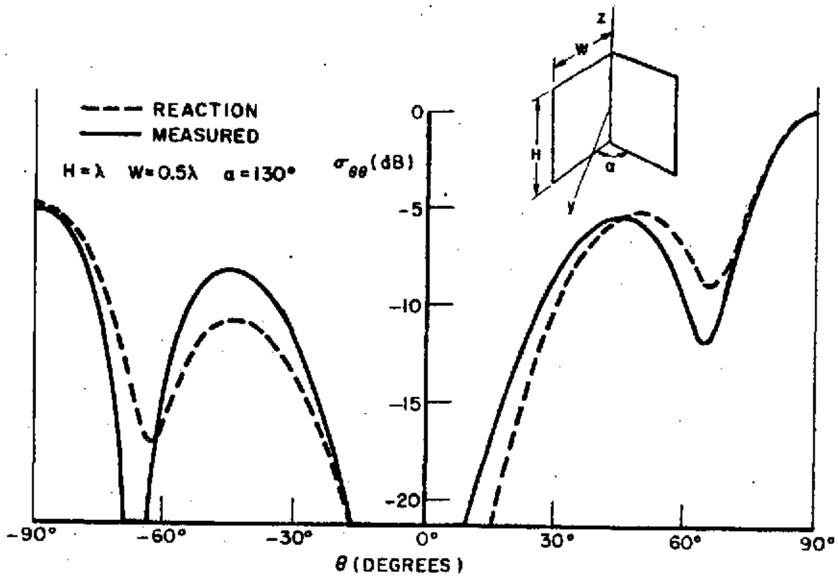

Fig. 44--Normalized backscatter cross-section in the yz plane of a corner reflector. $\sigma_{\theta\theta}(\theta, \phi)$ = -.38 dB at (90°,90°).

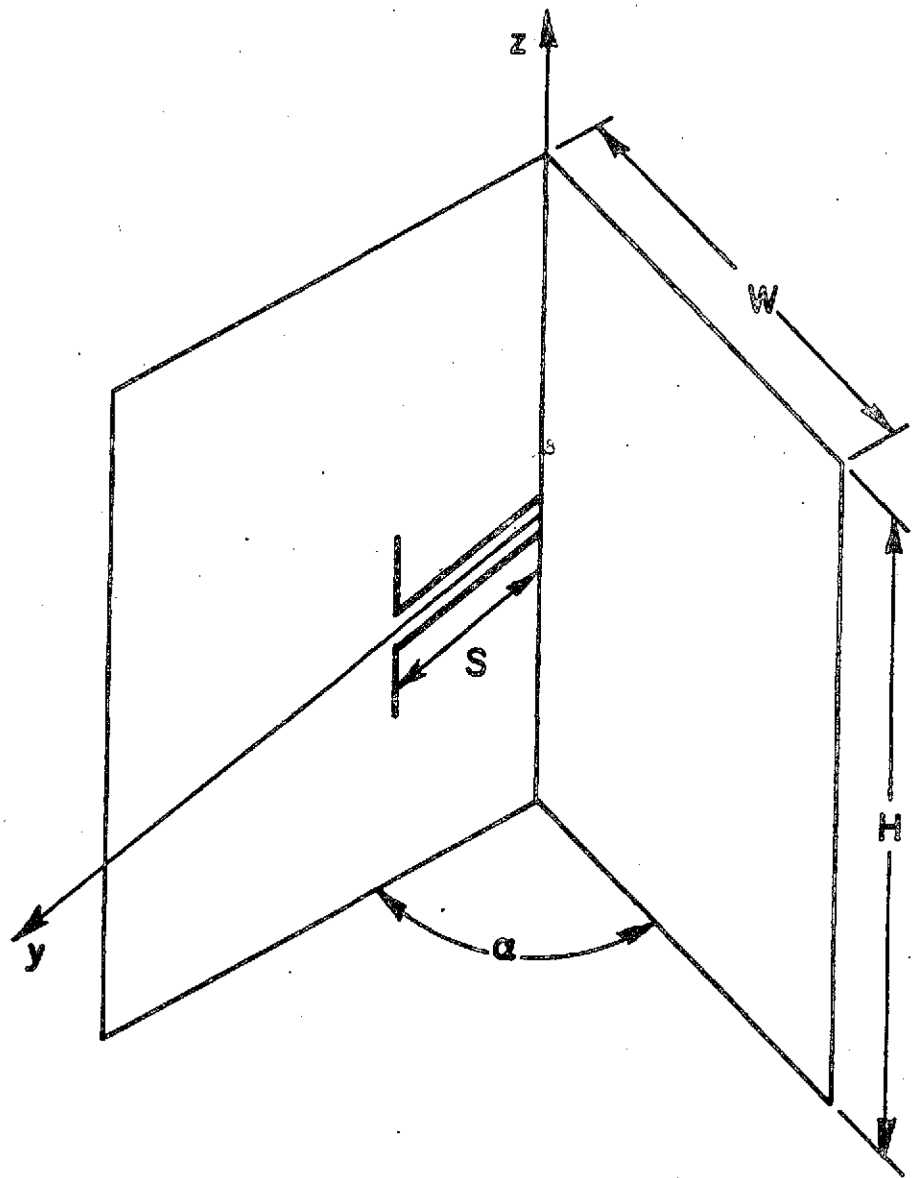

Fig. 45--Corner-reflector antenna.

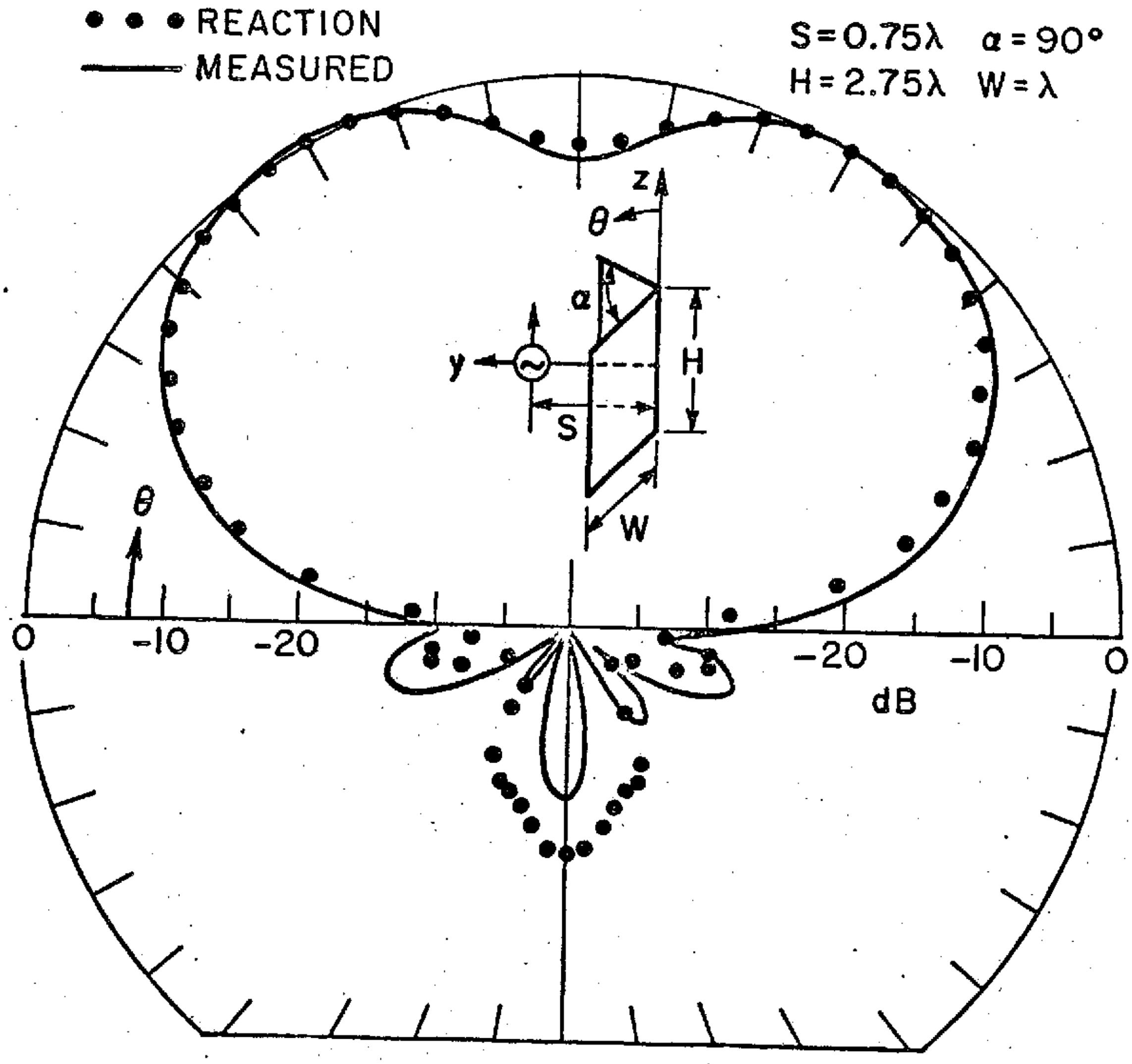

Fig. 46--Relative gain in the E-plane of a corner-reflector antenna. G(θ, φ) = 4.31 dB at (90°,90°).

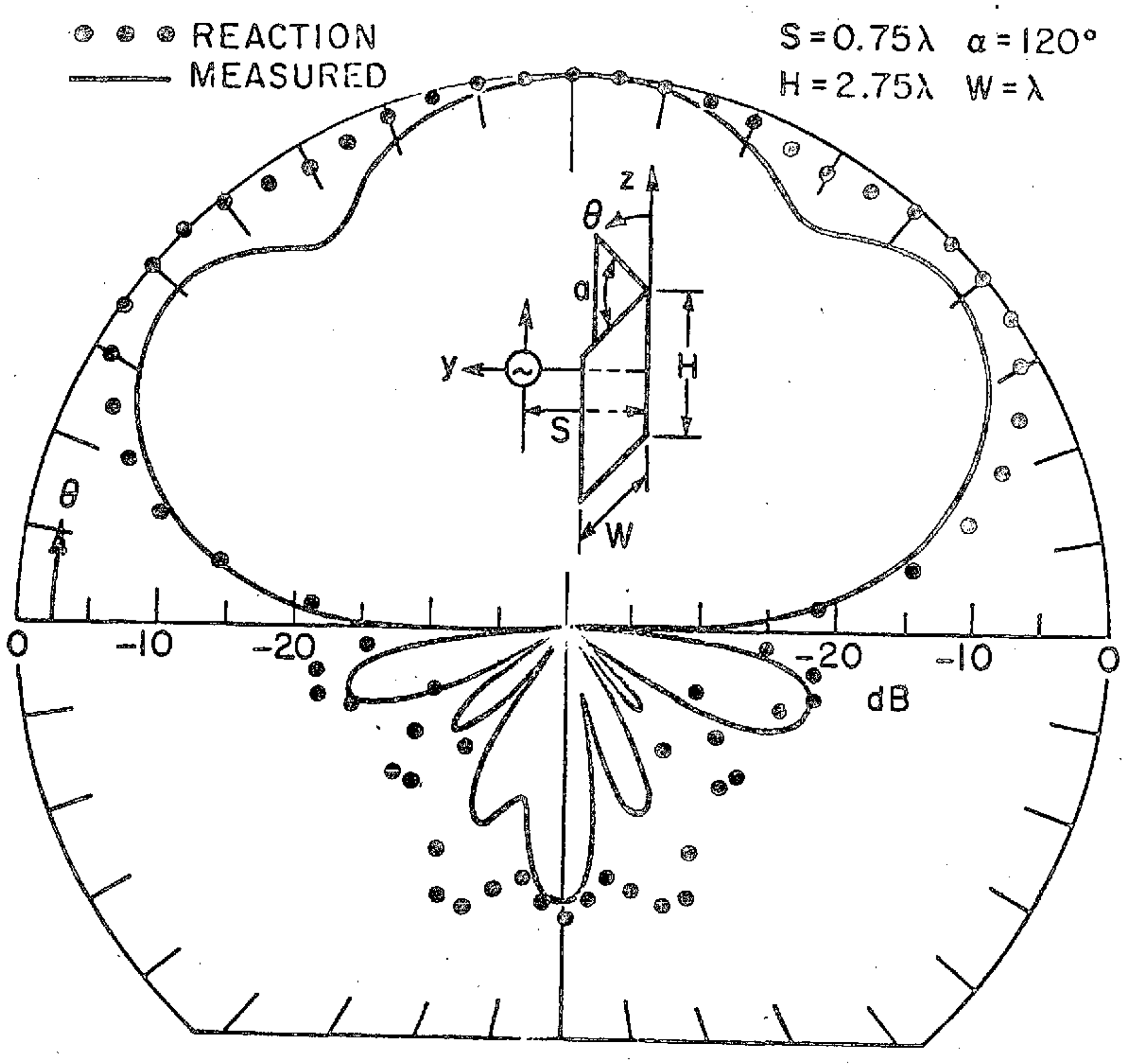

Fig. 47--Relative gain in the E-plane of a corner-reflector antenna. $G(\theta, \phi) = 4.05$ dB at $(90^\circ, 90^\circ)$.

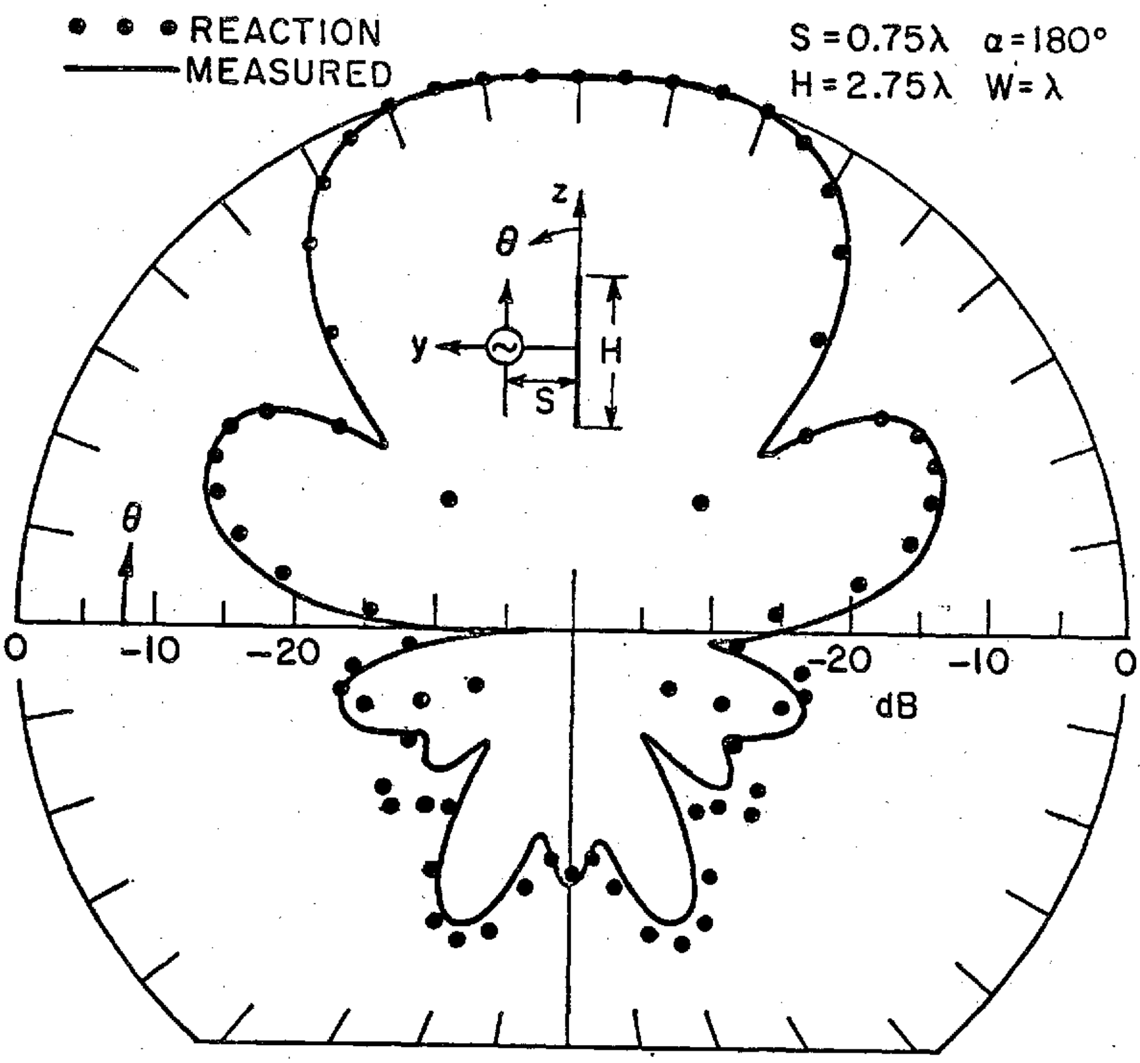

Fig. 48--Relative gain in the E-plane of a corner-reflector antenna. G(θ, ϕ) = 7.48 dB at (90°,90°).

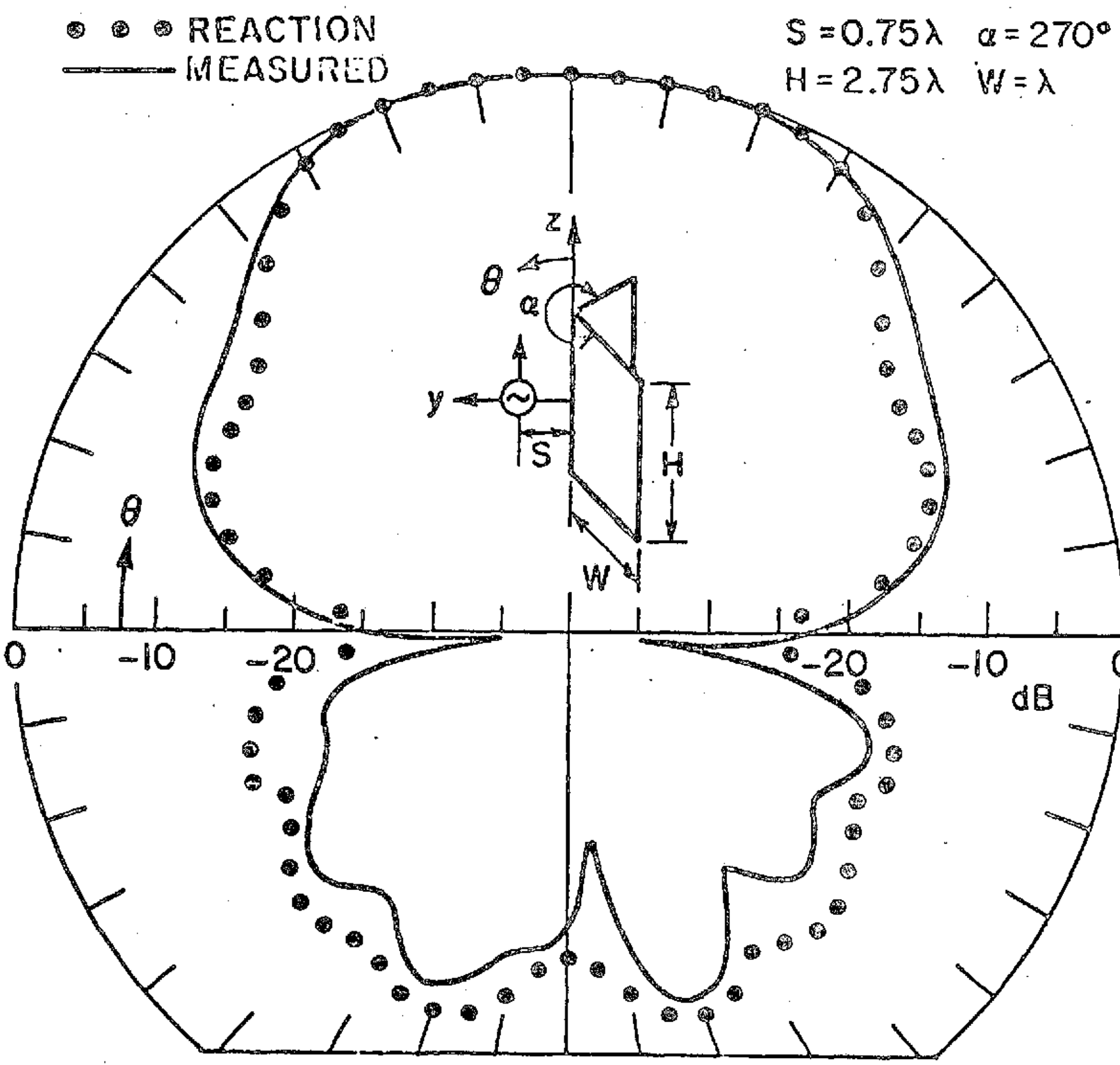

Fig. 49--Relative gain in the E-plane of a corner-reflector antenna. $G(\theta, \phi) = -1.06$ dB at $(90°, 90°)$.

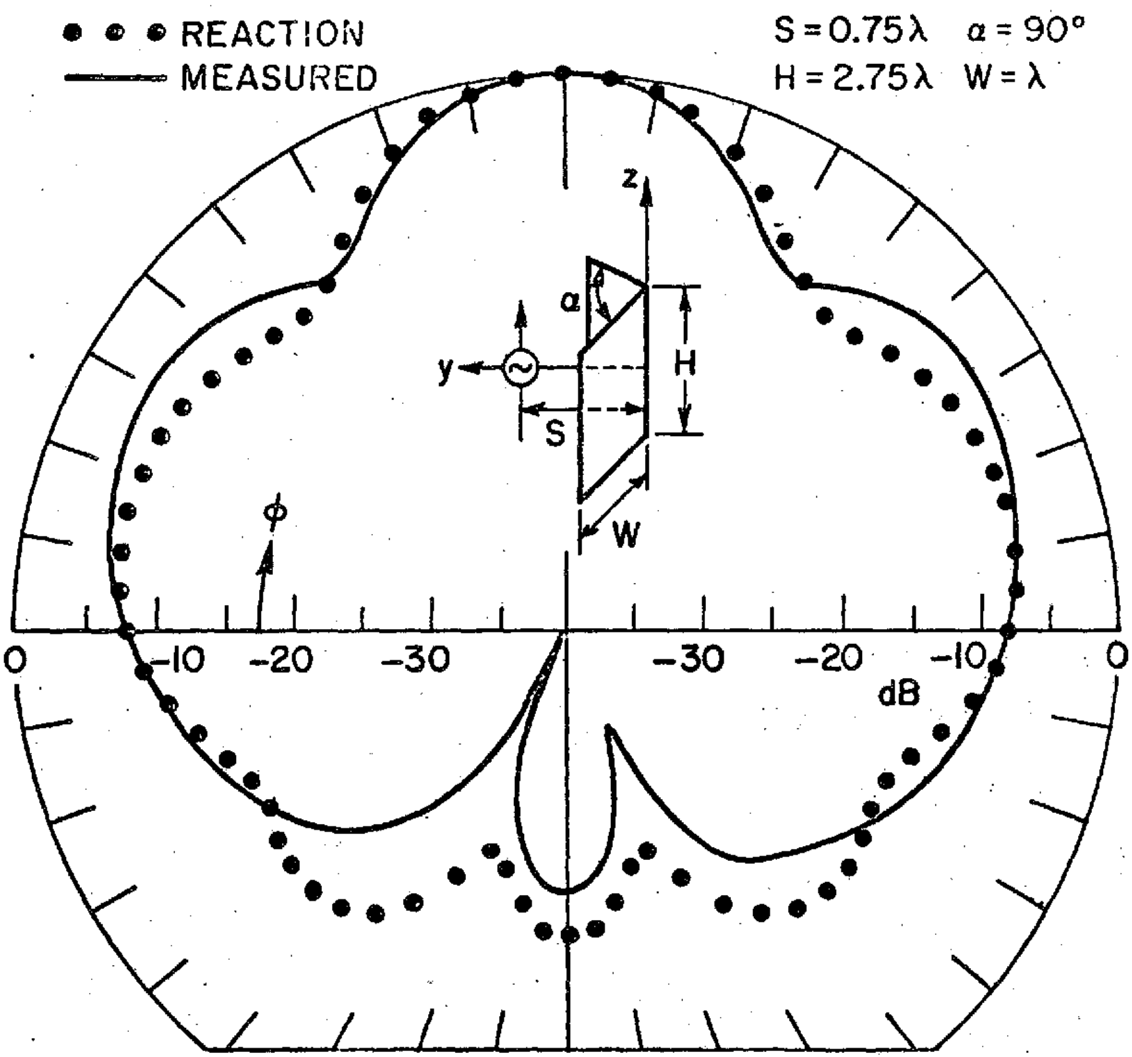

Fig. 50--Relative gain in the H-plane of a corner-reflector antenna. $G(\theta, \phi)$ = 4.31 dB at $(90^\circ, 90^\circ)$.

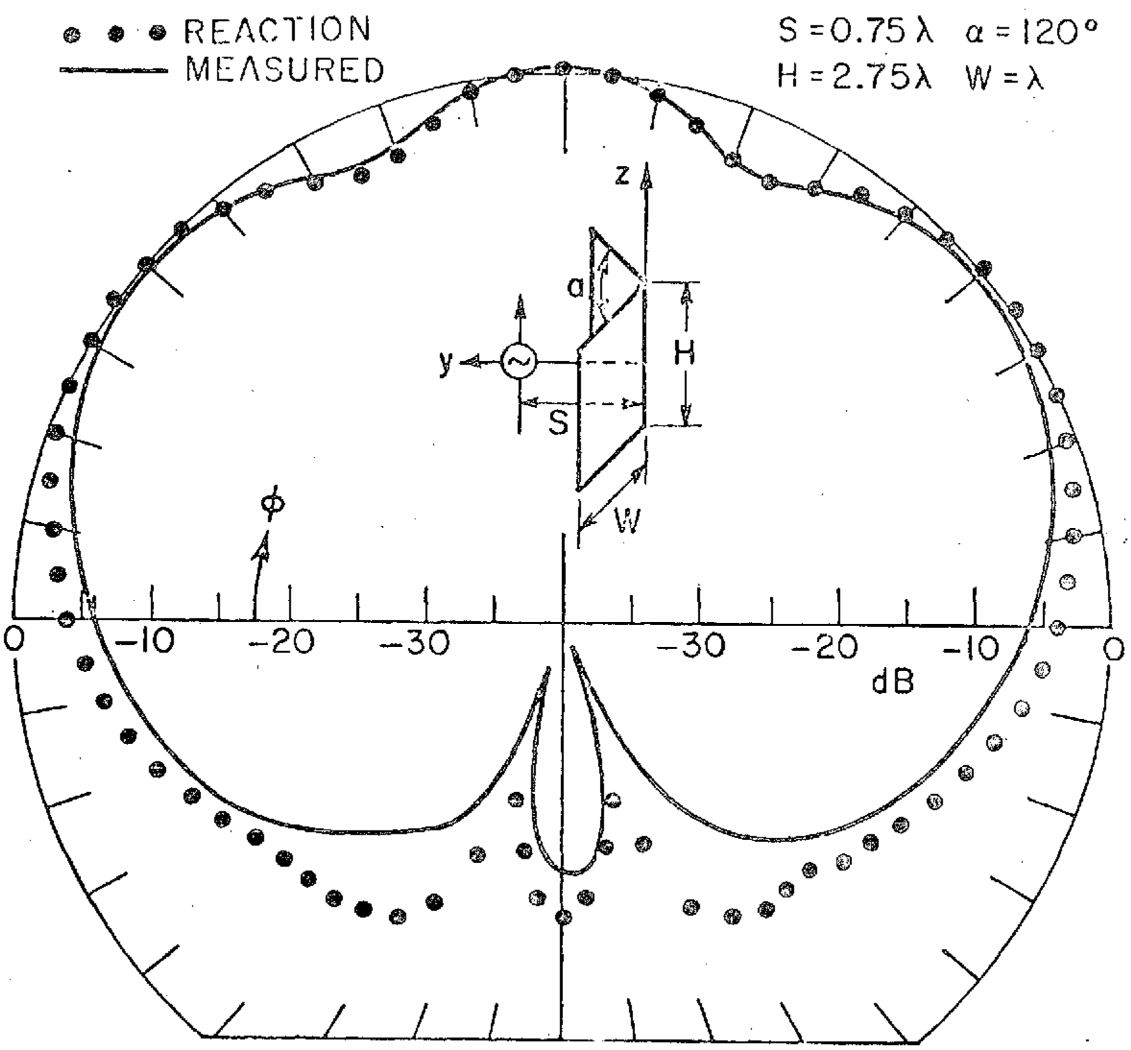

Fig. 51--Relative gain in the H-plane of a corner-reflector antenna. G(θ, φ) = 4.05 dB at (90°,90°).

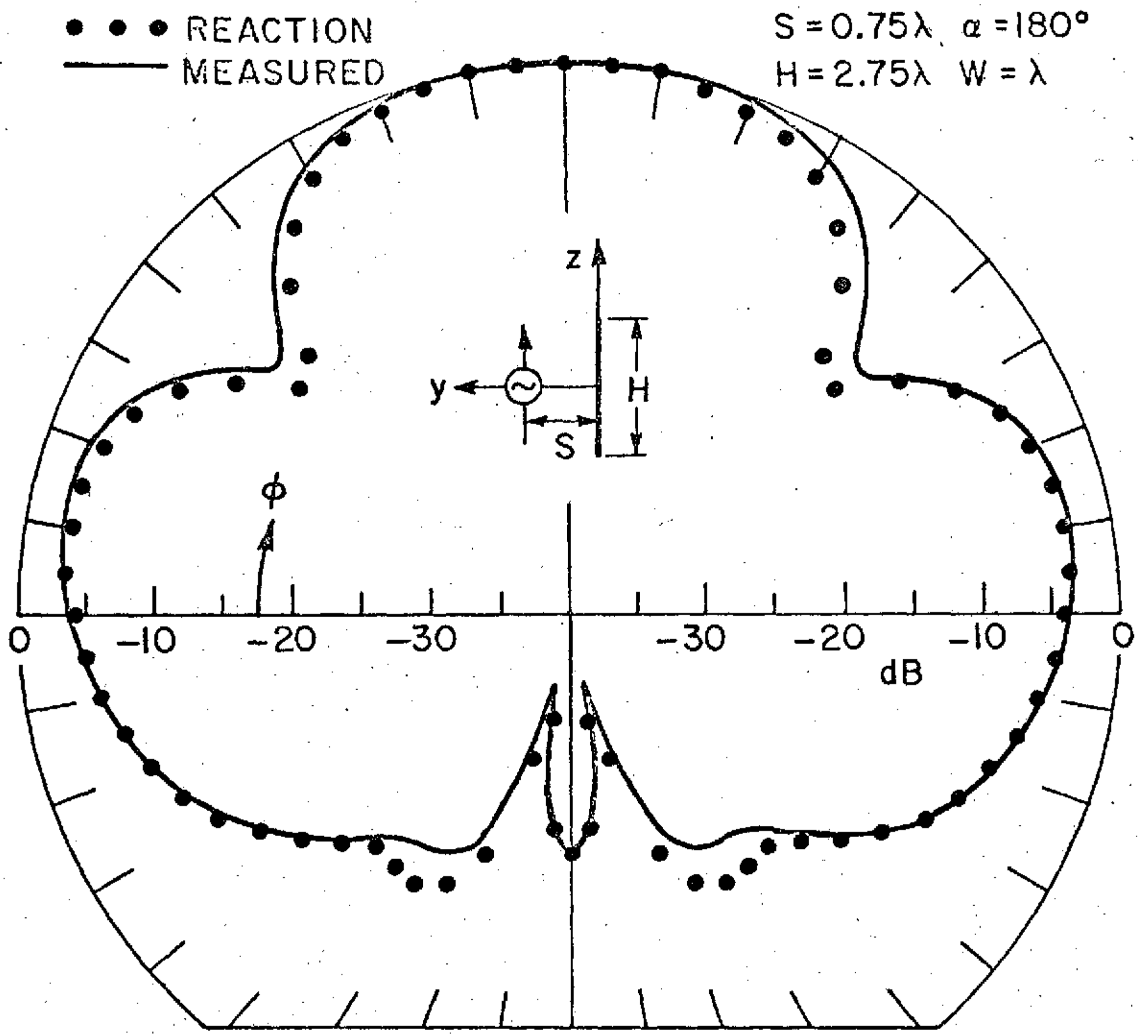

Fig. 52--Relative gain in the H-plane of a corner-reflector antenna. $G(\theta, \phi)$ = 7.48 dB at (90°,90°).

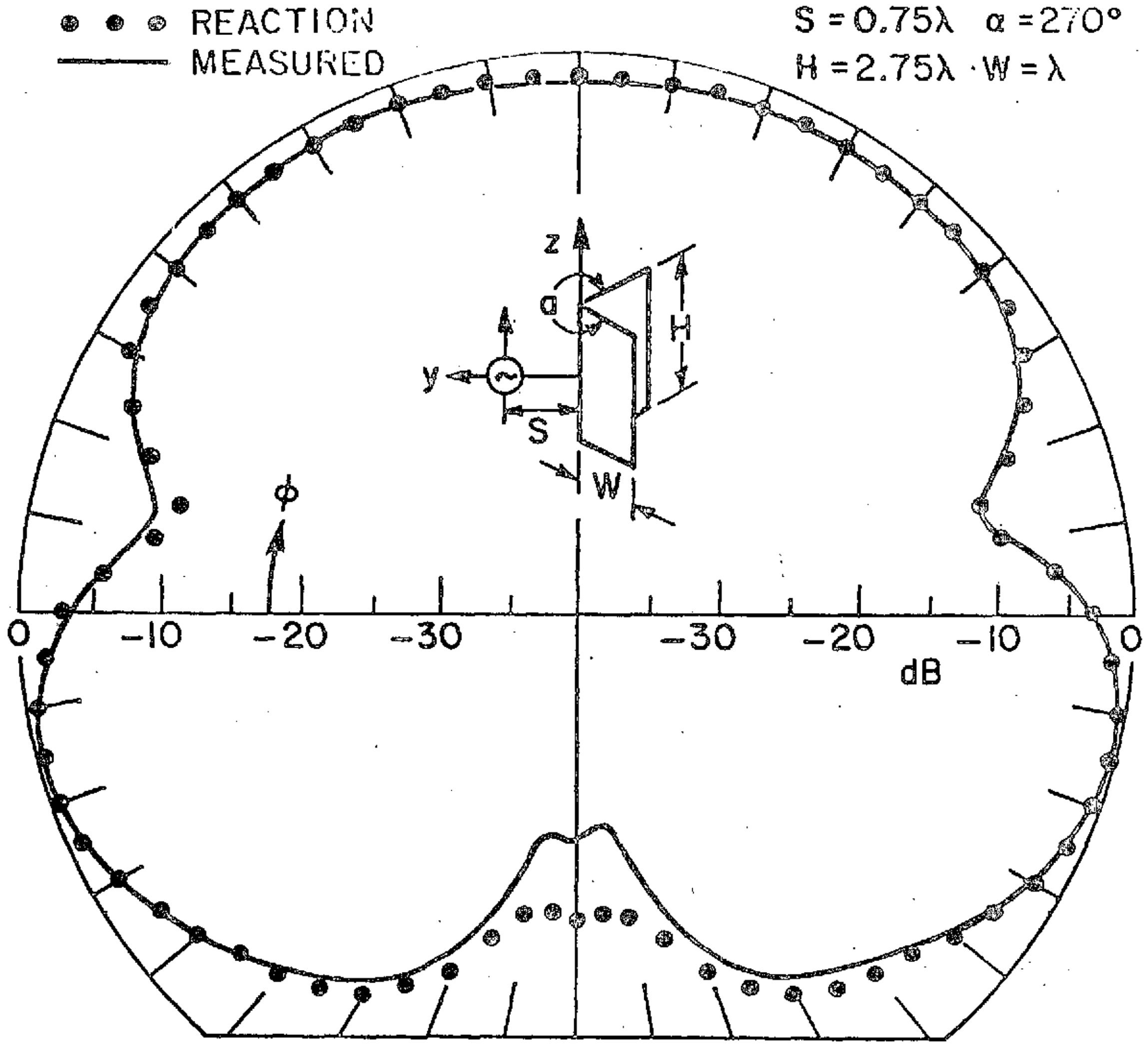

Fig. 53--Relative gain in the H-plane of a corner-reflector antenna. $G(\theta, \phi) = -1.06$ dB at $(90^\circ, 90^\circ)$.

CHAPTER IX
SUMMARY AND DISCUSSIONS

The reaction concept and Galerkin's method are employed to develop an integral-equation formulation for radiation and scattering from perfectly-conducting plates, corner reflectors and dielectric-coated conducting cylinders.

For the two-dimensional problems, the contour of the cylinder is divided into segments and the surface-current density on the conducting surface is expanded with sinusoidal bases for the TE polarization and rectangular-pulse bases for the TM polarization. Reaction tests are enforced with electric test sources. This reaction-Galerkin technique yields accurate results for scattering by cylinders with as few as five segments per wavelength. Furthermore, this technique provides a symmetric impedance matrix in the matrix equation. The point-matching procedure, on the other hand, generates an unsymmetric impedance matrix and requires on the order of ten segments per wavelength to yield accurate results.

For a coated cylinder, the dielectric layer is modeled with the equivalent polarization current radiating in free space. Maxwell's equations and the boundary conditions are employed to express the polarization-current distribution in terms of the surface-current density on the conducting surface. The impedance matrix has the same size for an uncoated cylinder and a cylinder with a thin dielectric coating. It is found that the polarization-current model is more accurate than the popular surface-impedance model.

For the three-dimensional problems, a new model that involves dividing the conducting surface into cells, expanding the current distribution with subsectional bases and enforcing reaction tests with electric surface dipoles is developed and applied to the problems of radiation and scattering from perfectly-conducting rectangular plates and corner reflectors up to six square wavelengths in size. For arbitrary aspect and polarization, this sinusoidal-Galerkin technique yields good results for scattering by a one-wavelength square plate with an 18 x 18 matrix. For this same problem, the wire-grid model (and the surface-cell model with pulse bases and collocation) requires a matrix size of 60 x 60. Thus, the new surface-current model offers a substantial improvement in computer storage requirements. This model can also be applied to nonplanar surfaces provided that the general surface is approximated by a set of planar cells. In this case, the cells may have to be reduced in size to obtain a satisfactory fit. This will increase the number of cells employed for modeling, thereby limiting the application of this model for nonplanar surfaces. However, extensions can be made such that the individual cell is doubly-curved in nature, and integration techniques can

be developed for evaluating the reaction between these curved cells. With this extension, the problem of radiation and scattering from curved surfaces can be analyzed more efficiently, and bodies with surface areas up to ten square wavelengths may be analyzed. For problems involving more complicated geometry, such as antennas mounted on aircraft, a combination of planar and curved cells can be employed to model the aircraft surfaces.

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APPENDIX A

ELECTRIC FIELD INDUCED IN THE THIN DIELECTRIC LAYER COATED ON A PERFECTLY-CONDUCTING POLYGON CYLINDER ILLUMINATED BY AN INCIDENT PLANE WAVE

Consider a dielectric-coated, perfectly-conducting polygon cylinder illuminated by an incident plane wave as shown below. The dielectric layer is a source-free region and has a thickness of d and a dielectric constant ϵ. The segment length of the cylinder is denoted by ℓ. Define a coordinate system such that $\hat{\zeta} \times \hat{t} = \hat{z}$,

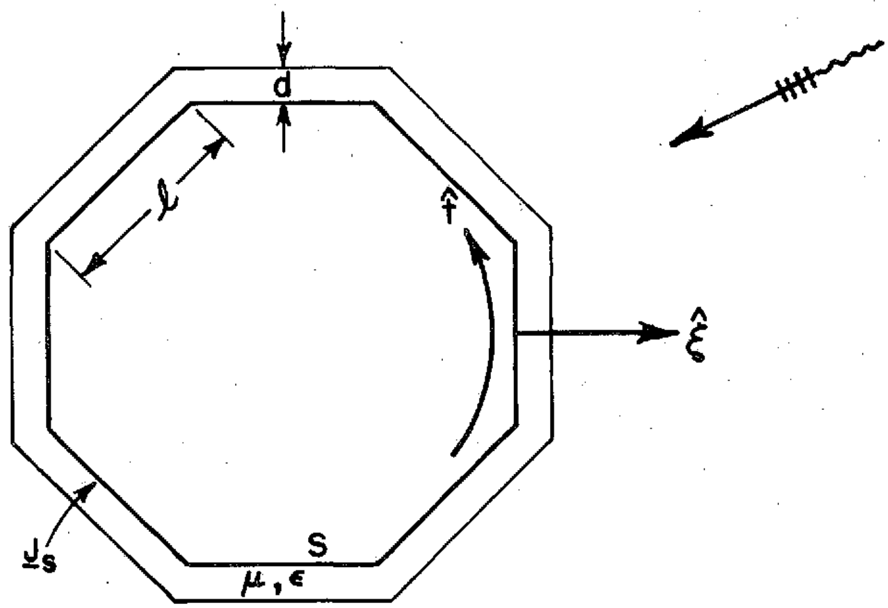

Fig. 54--Dielectric-coated, perfectly-conducting polygon cylinder illuminated by an incident plane wave.

where $\hat{t}$ is a unit vector tangent to the conducting surface and $\hat{\zeta}$ is a unit vector normal to the conducting surface. The incident plane wave is either transverse-electric or transverse-magnetic with respect to $\hat{z}$-axis.

For a perfectly-conducting cylinder with a thin dielectric coating ($d/\ell \ll 1$), the magnetic field inside the dielectric layer can be expressed as follows:

(103) $\underline{H} = \underline{H}_S \cos(\beta\zeta)$

where $\underline{H}_S$ is the magnetic field induced on the conducting surface S and is related to the surface-current density $\underline{J}_S$ by

(104) $\underline{H}_S = \underline{J}_S \times \hat{\zeta}.$

Coordinate ζ measures the distance normally outward from the conducting surface and β is the transverse propagation constant in the dielectric region which can be determined from the wave equation. From Eqs. (103) and (104) and Maxwell's equations, the electric field inside the dielectric layer can be given as

(105) $\underline{E} = \frac{1}{j\omega\varepsilon} \nabla \times \left[(\underline{J}_S \times \hat{\zeta}) \cos(\beta\zeta)\right].$

For the transverse-magnetic incidence case, the surface current density has only a $\hat{z}$-component $\underline{J}_S = \hat{z}\, J_S(t)$ and $\beta = k_1 = k\sqrt{\varepsilon_r}$. From Eq. (105), it can be shown that

(106) $\underline{E} = \frac{-k_1}{j\omega\varepsilon} \sin(k_1\zeta)\, \underline{J}_S$ (TM case)

For transverse-electric case, the surface-current density has only a $\hat{t}$-component $\underline{J}_S = \hat{t}\, J_S(t)$ and $\beta = k_\zeta = k\sqrt{\varepsilon_{r-1}}$. For this case, Eq. (105) yields

(107) $\underline{E} = \frac{1}{j\omega\varepsilon} \left[(\hat{z} \times \underline{J}_S') \cos(k_\zeta\zeta) + k_\zeta \sin(k_\zeta\zeta)\, \underline{J}_S\right].$

If $k_\zeta\zeta << 1$, Eq. (107) reduces to

(108) $\underline{E} = \frac{1}{j\omega\varepsilon} \cos(k_\zeta\zeta)\, (\hat{z} \times \underline{J}_S')$ (TE case)

where $\underline{J}_S'$ is the derivative of the surface-current density.

APPENDIX B
COMPUTER PROGRAMS FOR RADIATION AND SCATTERING
FROM TM DIELECTRIC-COATED CYLINDERS

```
      INCLUDE CROUTB,2902W
      COMPLEX C(40,40),ZS,CJ(40)
      COMPLEX EJ,EJJ,CST,EM,EJE
      DIMENSION X(40),Y(40),D(40),IA(40),IB(40)
      COMMON/COA/CCC,ER2,TSK2L
      DATA IDM,INT/40,10/
      DATA  PI,TP/3.14159,6.28318/
      WRITE(1,998)
998   FORMAT(5X,'LOP=?,ZS=?/')
      READ(0,-) LOP,ZS
10    CONTINUE
      WRITE(1,999)
999   FORMAT(5X,'NCASE=? 1=RECTANGULAR,2=CIRCULAR,3=STRIP/')
      READ(0,-) NCASE
      WRITE(1,997)
      READ(0,-) PHI,DPH,BSC
      GO TO (400,500,300),NCASE
300   CONTINUE
      PHI=90.
      READ(0,-) WK,NM,TSL,ER2
      TSK2L=TSL*TP*SQRT(ER2)
      CCC=(ER2-1.)/ER2*(1.-COS(TSK2L))
      WRITE(6,4) WK
      WK=WK*2.
      NP=NM+1
      DX=WK/NM
      DO 1 I=1,NP
      Y(I)=.0
      X(I)=DX*(I-1)
1     CONTINUE
      DO 2 J=1,NM
      IA(J)=J
      IB(J)=J+1
      D(J)=DX
      CJ(J)=CMPLX(2.*PI/DX,.0)
2     CONTINUE
      GO TO 600
500   CONTINUE
      PHI=.0
      PHR=PHI*PI/180.
      CPH=COS(PHR)
      SPH=SIN(PHR)
      CST=TP/(-30.*(.707,-.707))
      READ(0,-) DL,TSL,ER2,XKS
      WRITE(6,4) DL,TSL,ER2
```

```
CC
      TSK2L=TSL*TP*SQRT(ER2)
      CCC=(ER2-1.)/ER2*(1.-COS(TSK2L))
CC
      NM=12+20.*DL
      IF(NM.LT.16) NM=16
      WRITE(1,-)NM
      NP=NM
      PHO=TP/NM
      BL=.5*(PI*DL/NM)/SIN(.5*PHO)
      DO 11 I=1,NM
      X(I)=TP*BL*COS(PHO*(I-1))
      Y(I)=TP*BL*SIN(PHO*(I-1))
      IA(I)=I
      IB(I)=I+1
      IF(I.EQ.NM) IB(I)=1
      D(I)=TP*BL*2.*SIN(.5*PHO)
 11   CONTINUE
      GO TO 80
 400  CONTINUE
 997  FORMAT(5X,'PHI,DPH,BSC=?,BSC>0. **YES/')
      IF(LOP.EQ.2) READ(0,-) XKS,YKS
      IF(LOP.EQ.2) WRITE(6,4)XKS,YKS
      PHR=PHI*PI/180.
      CPH=COS(PHR)
      SPH=SIN(PHR)
      CST=TP/(-30.*(.707,-.707))
      READ(0,-) AX,BY,TSL,ER2
      WRITE(6,4) AX,BY,TSL,ER2
      TSK2L=TSL*Tp*SQRT(ER2)
      CCC=(ER2-1.)/ER2*(1.-COS(TSK2L))
      CALL REC(AX,BY,IDM,NM,NP,X,Y,IA,IB)
      DO 260 I=1,NM
      X(I)=TP*X(I)
      Y(I)=TP*Y(I)
 260  CONTINUE
      DO 45 J=1,NM
      K=IA(J)
      L=IB(J)
      D(J)=SQRT((X(L)-X(K))**2+(Y(L)-Y(K))**2)
 45   CONTINUE
 80   CONTINUE
      DO 22 I=1,NM
      KA=IA(I)
      KB=IB(I)
      IF(LOP.EQ.2) GO TO 33
      CALL CFF(X(KA),Y(KA),X(KB),Y(KB),D(I),CPH,SPH,ZS,EJ,EM,EJE)
      CJ(I)=EJ*CST/D(I)/D(I)
CCC
      GO TO 22
```

```
 33   CONTINUE
      CALL CELS(X(KA),Y(KA),X(KB),Y(KB),XKS,YKS,D(I),INT,EJ)
      CJ(I)=EJ*TP*TP/D(I)/D(I)
      CJ(I)=CJ(I)/(-60.*PI)
CCC
      WRITE(6,4) CJ(I)
 22   CONTINUE
 600  CONTINUE
      ISYM=0
C
      RS=CABS(ZS)
      IF(RS.GT.0.) ISYM=1
C
CC
      IF(TSL.GT.0.) ISYM=1
CC
      CALL CDANT(C,D,X,Y,ZS,IA,IB,ISYM,IDM,INT,NM,NP)
      IF(ISYM.EQ.10) GO TO 1000
      DO 3 I=1,1
      DO 3 J=1,NM
      WRITE(6,4) C(I,J)
 4    FORMAT(5X,5F10.4/)
 3    CONTINUE
      CALL CROUT(C,CJ,NM,IDM,ISYM,1,1)
      PH=0.0
      IF(BSC.GT.0.) PH=PHI
 30   CONTINUE
      PHR=PH*PI/180.
      CPH=COS(PHR)
      SPH=SIN(PHR)
      EJJ=(.0,.0)
      DO 250 K=1,NM
      KA=IA(K)
      KB=IB(K)
      CALL CFF(X(KA),Y(KA),X(KB),Y(KB),D(K),CPH,SPH,
     2ZS,EJ,EM,EJE)
      EJJ=EJJ+(EJ+EJE)*CJ(K)+EM*CJ(K)
 250  CONTINUE
      IF(LOP.EQ.2) EJJ=EJJ+CEXP(CMPLX(.0,XKS*CPH+YKS*SPH+PI/4.))
      EAB=CABS(EJJ)
      EWL=2.*PI*EAB*EAB
      EWS=EWL*PI*2.
      WRITE(1,-) PH,EWL
      WRITE(6,4) PH,EWL,EAB
      PH=PH+DPH
      PHEND=360.
      IF(BSC.GT.0.) PHEND=PHI
      IF(PH.LE.PHEND) GO TO 30
 1000 CONTINUE
      READ(0,-) IC
      IF(IC.EQ.0) GO TO 10
      CALL EXIT
      END
```

```
      SUBROUTINE CDANT(C,D,X,Y,ZS,IA,IB,ISYM,IDM,INT,NM,NP)
      COMPLEX ZS,P11,C(IDM,IDM)
      DIMENSION X(IDM),Y(IDM),D(IDM),IA(IDM),IB(IDM)
      DO 20 I=1,NM
      DO 20 J=1,NM
   20 C(I,J)=(.0,.0)
      DMAX=.0
      DO 25 J=1,NM
      DK=D(J)
   25 IF(DK.GT.DMAX) DMAX=DK
      WRITE(1,-)DMAX
      IF(DMAX.LT.3.) GO TO 30
      ISYM=10
      RETURN
   30 CONTINUE
      DO 200 K=1,NM
      KA=IA(K)
      KB=IB(K)
      DK=D(K)
      LL=1
      IF(ISYM.EQ.0) LL=K
      DO 200 L=LL,NM
      LA=IA(L)
      LB=IB(L)
      DL=D(L)
      IF(K.EQ.L) GO TO 120
      IND=(LA-KA)*(LB-KA)*(LA-KB)*(LB-KB)
      IF(IND.EQ.0) GO TO 80
C
      CALL ZMMC(X(KA),Y(KA),X(KB),Y(KB),X(LA),Y(LA),X(LB),Y(LB),
     2ZS,DK,DL,INT,P11)
      GO TO 168
   80 CONTINUE
      JM=KB
      JC=KA
      IND=(KB-LA)*(KB-LB)
      IF(IND.NE.0) GO TO 82
      JC=KB
      JM=KA
   82 JP=LA
      IF(LB.EQ.JC) GO TO 83
      JP=LB
   83 CALL ZMMB(X(JM),Y(JM),X(JC),Y(JC),X(JP),Y(JP),ZS,DK,DL,INT,P11)
      GO TO 168
  120 CALL ZMMA(DK,ZS,P11)
  168 C(K,L)=P11
  200 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE ZMMA(DK,ZS,P11)
      COMPLEX ZS,H0,H1,VH,GI,Y,P11
      COMMON/COA/CCC,ER2,TSK2L
      DATA  PI/3.14159/
      TP=2.*PI
      ETA=120.*PI
      CALL HANK(DK,H0,H1,1)
      GI=VH(DK)
      D=DK/TP
      P11=D/(2.*TP)*GI-1./(2.*TP*TP)*(DK*H1-(.0,2.)/PI)
      P11=P11*TP*ETA/D/D
      Y=P11*2./ETA
CC
      P11=P11+P11*CCC
CC
C
      P11=P11+ZS*PI/DK
C
      RETURN
      END
      SUBROUTINE ZMMP(X1,Y1,X2,Y2,X3,Y3,ZS,
     2DK1,DK2,INT,Q11)
      COMPLEX VH,ZS,Q11,Y11,CCP,G1,G2,G12,
     2H10,H11,H20,H21,H120,H121,HP0,HP1,HM0,HM1
      COMPLEX DQ11P,DQ11M,RKH1P,RKH1M,DQ11,DCNT
      COMMON/COA/CCC,ER2,TSK2L
      DATA  CCP/(.0,.63662)/
      DATA  PI/3.14159/
      RS=CABS(ZS)
      CBET=(X2-X1)/DK1
      SBET=(Y2-Y1)/DK1
      XB=(X3-X1)*CBET+(Y3-Y1)*SBET
      YB=-(X3-X1)*SBET+(Y3-Y1)*CBET
      CAL=(XB-DK1)/DK2
      SAL=ABS(YB/DK2)
      AL=ATAN2(SAL,CAL)
      CNT=15.*4.*PI*PI/DK2/DK1
      DK12=DK1+DK2
      IF(CAL.LT.0.) GO TO 20
      IF(SAL.GT.0.04) GO TO 20
      CALL HANK(DK1,H10,H11,1)
      CALL HANK(DK2,H20,H21,1)
      DK12=DK1+DK2
      CALL HANK(DK12,H120,H121,1)
      G1=VH(DK1)
      G2=VH(DK2)
      G12=VH(DK12)
      Q11=CNT*(-CCP+DK1*(H11-G1)+DK2*(H21-G2)-DK12*(H121-G12))
CC
      Q11=Q11+Q11*CCC
CC
```

```
      RETURN
   20 CONTINUE
      INP=2*(INT/2)
      IP=INP+1
      JP=IP
      FIT=INP
      ALT=AL/2.
      CALT=COS(ALT)
      SALT=SIN(ALT)
      RCP=DK12*CALT
      RSP=(DK2-DK1)*SALT
      PHC=ATAN2(RSP,RCP)
      SGI=-1.
      PHM=-ALT
      PHP=PHC
      DPHM=(PHC+ALT)/FIT
      DPHP=(ALT-PHC)/FIT
      Y11=(.0,.0)
C
      DQ11=(.0,.0)
C
      DO 200 I=1,IP
      D=SGI+3.
      IF(I.EQ.1.OR.I.EQ.IP ) D=1.
      SAP=SIN(ALT+PHP)
      SAM=SIN(ALT-PHM)
      ARGP=DK2*SAL/SAP
      ARGM=DK1*SAL/SAM
      CALL HANK(ARGP,HP0,HP1,1)
      CALL HANK(ARGM,HM0,HM1,1)
C
      IF(RS.LE.0.) GO TO 300
      DARGP=ARGP/FIT
      DARGM=ARGM/FIT
      RKP=.0
      RKM=.0
      SGJ=-1.
      DQ11P=(.0,.0)
      DQ11M=(.0,.0)
      RKH1P=CCP
      RKH1M=CCP
      DO 100 J=1,JP
      C=SGJ+3.
      IF(J.EQ.1.OR.J.EQ.JP) C=1.
      IF(J.EQ.1) GO TO 94
      CALL HANK(RKP,H10,H11,1)
      CALL HANK(RKM,H20,H21,1)
      RKH1P=RKP*H11
      RKH1M=RKM*H21
```

```
 94   CONTINUE
      DQ11P=DQ11P+RKH1P*C
      DQ11M=DQ11M+RKH1M*C
      RKP=RKP+DARGP
      RKM=RKM+DARGM
      SGJ=-SGJ
 100  CONTINUE
      DQ11=DQ11+SAM*(DQ11P*DARGP*DPHP+DQ11M*DARGM*DPHM)*D
 300  CONTINUE
C
      Y11=Y11+(ARGM*HM1*DPHM+ARGP*HP1*DPHP)*D
      PHM=PHM+DPHM
      PHP=PHP+DPHP
      SGI=-SGI
  200 CONTINUE
      Q11=CNT*(Y11/3./SAL-CCP*AL/SAL)
CC
      Q11=Q11+Q11*CCC
CC
C
      DCNT=ZS*CMPLX(.0,-2.*PI)/(4.*DK1*DK2*SAL)
      DQ11=DQ11*DCNT/9.
      Q11=Q11+DQ11
C
      RETURN
      END
      SUBROUTINE ZMMC(X1,Y1,X2,Y2,X3,Y3,X4,Y4,ZS,
     2DK1,DK2,INT,P11)
      COMPLEX P11,G3,G13,G23,G123,VH,HA0,HA1,HB0,HB1,HC0,HC1,HD0,HD1,
      COMPLEX HX,HY,DP11,DCNT
      COMPLEX H0,H1
      COMMON/COA/CCC,ER2,TSK2L
      DATA  PI/3.14159/
      RS=CABS(ZS)
      CBET=(X2-X1)/DK1
      SBET=(Y2-Y1)/DK1
      XA=(X3-X1)*CBET+(Y3-Y1)*SBET
      XB=(X4-X1)*CBET+(Y4-Y1)*SBET
      YA=-(X3-X1)*SBET+(Y3-Y1)*CBET
      YB=-(X4-X1)*SBET+(Y4-Y1)*CBET
      CAL=(XB-XA)/DK2
      SAL=(YB-YA)/DK2
      CNT=15.*4.*PI*PI/DK1/DK2
      ASAL=ABS(SAL)
      IF(ASAL.GT..04) GO TO 20
      IF(YB.NE..0) GO TO 20
      DK3=ABS(XA-DK1)
C
```

```
      IF(XA.LT.0.0) DK3=ABS(XB)
C
      DK13=DK1+DK3
      DK23=DK2+DK3
      DK123=DK1+DK2+DK3
      CALL HANK(DK3  ,HA0,HA1,1)
      CALL HANK(DK13 ,HB0,HB1,1)
      CALL HANK(DK23 ,HC0,HC1,1)
      CALL HANK(DK123,HD0,HD1,1)
      G3=VH(DK3)
      G13=VH(DK13)
      G23=VH(DK23)
      G123=VH(DK123)
      P11=CNT*(DK3*(G3-HA1)-DK13*(G13-HB1)-DK23*(G23-HC1)+DK123*
     2(G123-HD1))
CC
      P11=P11+P11*CCC
CC
      RETURN
   20 CONTINUE
      RMIN=10000.
      X=XA
      Y=YA
      DX=DK2*CAL/4.
      DY=DK2*SAL/4.
      DO 40 J=1,5
      YS=.0
      R=ABS(Y)
      IF(R.GT.1.E-15)YS=Y*Y
      XS=0.0
      XAB=ABS(X-DK1)
      IF(XAB.GT.1.E-15) XS=XAB*XAB
      IF(X.LT.0.) R=SQRT(X*X+YS)
      IF(X.GT.DK1) R=SQRT(XS+YS)
      IF(R.LT.RMIN) RMIN=R
      X=X+DX
      Y=Y+DY
   40 CONTINUE
      FNT=1+(4*INT)/10
      ISS=FNT*DK1/RMIN
      ISS=2*(ISS/2)
      IF(ISS.LT.2) ISS=2
      IF(ISS.GT.20) ISS=20
      FSS=ISS
      ISQ=ISS+1
      DS=DK1/FSS
      ITT=FNT*DK2/RMIN
      ITT=2*(ITT/2)
      IF(ITT.LT.2) ITT=2
      IF(ITT.GT.20) ITT=20
```

```
      FTT=ITT
      ITQ=ITT+1
      DT=DK2/FTT
      DX=DT*CAL
      DY=DT*SAL
      X=XA
      Y=YA
      SGJ=-1.
      P11=(.0,.0)
C
      DP11=(.0,.0)
C
      DO 200 J=1,ITQ
      D=SGJ+3.
      IF(J.EQ.1.OR.J.EQ.ITQ) D=1.
      XP=.0
      YS=.0
      YAB=ABS(Y)
      IF(YAB.GT.1.E-15) YS=YAB*YAB
      EZ=(.0,.0)
C
      HX=(.0,.0)
      HY=(.0,.0)
C
      SGI=-1.
      DO 100 I=1,ISQ
      C=SGI+3.
      IF(I.EQ.1.OR.I.EQ.ISQ) C=1.
      DELX=ABS(X-XP)
      DXS=.0
      IF(DELX.GT.1.E-15) DXS=DELX*DELX
      RK=SQRT(DXS+YS)
C
      SPH=Y/RK
      CPH=(X-XP)/RK
C
      CALL HANK(RK,H0,H1,0)
      EZ=EZ+H0*C
      XP=XP+DS
      SGI=-SGI
C
      IF(RS.LE.0.) GO TO 100
      HX=HX+H1*(-SPH)*C
      HY=HY+H1*CPH*C
C
  100 CONTINUE
      EZ=EZ*DS/3.
      P11=P11+EZ*D
C
      HX=HX*DS/3.
      HY=HY*DS/3.
      DP11=DP11+(HX*CAL+HY*SAL)*D
```

```
C
      SGJ=-SGJ
      X=X+DX
      Y=Y+DY
  200 CONTINUE
      P11=P11*DT/3.
      P11=P11*CNT
CC
      P11=P11+P11*CCC
CC
C
      DCNT=ZS*CMPLX(.0,-2.*PI)/(4.*DK1*DK2)
      DP11=DP11*DCNT*DT/3.
      P11=P11+DP11
C
      RETURN
      END
      SUBROUTINE HANK(X,H,H1,ID)
      COMPLEX H,H1
      DATA TSP/.63661977/
      IF(X.GT.3.)GO TO 100
      XLN=TSP*ALOG(X/2.)
      B=.0
      B1=.0
      Y=.0
      Y1=.0
      X1=X/3.
      X2=X1*X1
      IF(X1.LT..1)GO TO 60
      X4=X2*X2
      X6=X2*X4
      IF(X1.LT..3)GO TO 55
      X8=X2*X6
      X10=X2*X8
      X12=X2*X10
      B=.21E-3*X12-.39444E-2*X10+.444479E-1*X8
      Y=-.24846E-3*X12+.427916E-2*X10-.4261214E-1*X8
      B1=.1109E-4*X12-.31761E-3*X10+.443319E-2*X8
      Y1=.27873E-2*X12-.400976E-1*X10+.3123951*X8
   55 B=B-.3163866*X6+1.2656208*X4
      Y=Y+.25300117*X6-.74350384*X4
      B1=B1-.3954289E-1*X6+.21093573*X4
      Y1=Y1-1.3164827*X6+2.1682709*X4
   60 B=B-2.2499997*X2+1.
      Y=Y+.60559366*X2+.36746691+XLN*B
      B1=X*(B1-.56249985*X2+.5)
      Y1=(Y1+.2212091*X2-.6366198)/X+XLN*B1
      GO TO 200
  100 SW=SQRT(X)
```

```
      X1=3./X
      X2=X1*X1
      X3=X1*X2
      X4=X1*X3
      X5=X1*X4
      X6=X1*X5
      F=.79788456-.77E-6*X1-.55274E-2*X2-.9512E-4*X3+.137237E-2*X4
     2-.72805E-3*X5+.14476E-3*X6
      T=X-.78539816-.4166397E-1*X1-.3954E-4*X2+.262573E-2*X3
     2-.54125E-3*X4-.29333E-3*X5+.13558E-3*X6
      B=F*COS(T)/SW
      Y=F*SIN(T)/SW
      F=.79788456+.156E-5*X1+.1659667E-1*X2+.17105E-3*X3-.249511E-2*X4
     2+.113653E-2*X5-.20033E-3*X6
      T=X-2.3561945+.12499612*X1+.565E-4*X2-.637879E-2*X3+.74348E-3*X4
     2+.79824E-3*X5-.29166E-3*X6
      B1=F*COS(T)/SW
      Y1=F*SIN(T)/SW
  200 H=CMPLX(B,-Y)
      H1=CMPLX(B1,-Y1)
      RETURN
      END
      COMPLEX FUNCTION VH(X)
      DIMENSION A(8),B(8)
      COMPLEX G(51),H0,H1
      DATA G/
     2(.0,.0),(.19933,-.34570),(.39470,-.50952),
     2(.58224,-.59927),(.75834,-.63787),(.91973,-.63707),
     2(1.06356,-.60490),(1.18750,-.54783),(1.28982,-.47156),
     2(1.36940,-.38136),(1.42577,-.28219),(1.45913,-.17871),
     2(1.47029,-.07527),(1.46070,.02420),(1.43231,.11618),
     2(1.38757,.19766),(1.32928,.26620),(1.26056,.31997),
     2(1.18468,.35775),(1.10497,.37897),(1.02473,.38367),
     2(.94712,.37250),(.87502,.34665),(.81101,.30780),
     2(.75721,.25802),(.71531,.19972),(.68647,.13551),
     2(.67131,.06814),(.66993,.00036),(.68187,-.06517),
     2(.70622,-.12595),(.74160,-.17976),(.78628,-.22471),
     2(.83821,-.25931),(.89512,-.28253),(.95464,-.29377),
     2(1.01435,-.29295),(1.0719,-.28043),(1.12509,-.25702),
     2(1.17193,-.22393),(1.21075,-.18270),(1.24021,-.13516),
     2(1.25939,-.08335),(1.26778,-.02940),(1.26529,.02451),
     2(1.25227,.07626),(1.22947,0.12385),(1.19799,.16550),
     2(1.15928,.19969),(1.114997,.22523),(1.06701,.24129)/
      DATA PI/3.14159/
      DATA  A/.06233,.00404,.00101,.00054,
     2.00040,.00028,.00013,.00003/
      DATA  B/.79788,.01256,.00179,.00067,
     2.00041,.00025,.00011,.00002/
      IF(X.LE..9) GO TO 200
      IF(X.LT.10.) GO TO 100
```

```
      VJ=.0
      VY=.0
      DO 10 I=1,8
      K=I-1
      SIGN=(-1.)**I*(-1.)
      VJ=VJ+A(I)*(8./X)**(2*K+1)*SIGN
      VY=VY+B(I)*(8./X)**(2*K)*SIGN
10    CONTINUE
      VH=1.-CMPLX(VJ,-VY)*CEXP(CMPLX(.0,PI/4.-X))/SQRT(X)
      RETURN
100   CONTINUE
      Y=5.*X
      J=Y+1.5
      IF(J.LT.2) J=2
      IF(J.GT.50) J=50
      JM=J-1
      JP=J+1
      FJ=J
      YJ=FJ-1.
      Q=Y-YJ
      QT=Q/2.
      C=QT*(Q-1.)
      D=QT*(Q+1.)
      E=1.-Q**2
      VH=C*G(JM)+D*G(JP)+E*G(J)
      VH=CONJG(VH)
      RETURN
200   CONTINUE
      CALL HANK(X,H0,H1,2)
      HB0=X-X**3/9.+X**5/(9.*25.)-X**7/(9.*25.*49.)
      HB1=X*X/3.-X**4/45.+X**6/(63.*25.)-X**8/(81.*25.*49.)
      VH=X*H0+X*(HB0*H1-HB1*H0)
      RETURN
      END
      SUBROUTINE CFF(XA,YA,XB,YB,DK,CPH,SPH,ZS,EJ,EM,EJE)
      COMPLEX ZS,EJ,EJA,EJB,F,EM,EJE,E1,E2
      COMMON/COA/CCC,ER2,TSK2L
      DATA PI/3.14159/
      ETA=120.*PI
      CA=(XB-XA)/DK
      CB=(YB-YA)/DK
      A=XA*CPH+YA*SPH
      B=XB*CPH+YB*SPH
      EJA=CMPLX(COS(A),SIN(A))
      EJB=CMPLX(COS(B),SIN(B))
      C=CA*CPH+CB*SPH
      S=CB*CPH-CA*SPH
      F=CMPLX(.0,DK)*EJA
      IF(ABS(C).GT..001) F=(EJB-EJA)/C
      EJ=-30.*(.707,-.707)*F
      EM=ZS*(.707,-.707)*S*F/(4.*PI)
```

```
CC
      SP=1.+S/SQRT(FR2)
      SM=1.-S/SQRT(FR2)
      E1=CMPLX(O.,TSK2L)
      E2=CMPLX(O.,-TSK2L)
      IF(SP.GT.0.001) E2=(CEXP(CMPLX(.0,TSK2L*SP))-1.)/SP
      IF(SM.GT.0.001) E1=(CEXP(CMPLX(.0,-TSK2L*SM))-1.)/SM
      EJE=-FJ*(FR2-1.)/FR2*(E1+E2)*.5
CC
      RETURN
      END
      SUBROUTINE CELS(X1,Y1,X2,Y2,XS,YS,DK,INT,P1)
      COMPLEX HO,H1,P1
      DATA PI/3.14159/
      CBET=(X2-X1)/DK
      SBET=(Y2-Y1)/DK
      XA=(XS-X1)*CBET+(YS-Y1)*SBET
      YA=-(XS-X1)*SBET+(YS-Y1)*CBET
      X=XA
      Y=YA
      YSQ=Y**2
      RMIN=ABS(Y)
      IF(X.LT.0.0) RMIN=SQRT(X*X+YSQ)
      IF(X.GT.DK) RMIN=SQRT((X-DK)**2+YSQ)
      FNT=1+(4*INT)/10
      ISS=FNT*DK/RMIN
      ISS=2*(ISS/2)
      IF(ISS.LT.2) ISS=2
      FIT=ISS
      ISQ=ISS+1
      DS=DK/FIT
      SGI=-1.
      XP=0.
      P1=(0.,0.)
      DO 100 I=1,ISQ
      C=SGI+3.
      IF(I.EQ.1.OR.I.EQ.ISQ) C=1.
      DELX=X-XP
      RK=SQRT(DELX**2+YSQ)
      CALL HANK(RK,HO,H1,0)
      P1=P1+HO*C
      SGI=-SGI
      XP=XP+DS
  100 CONTINUE
      P1=P1*(-30.*PI)*DS/3.
      RETURN
      END
      SUBROUTINE REC(A,B,IDM,NM,NP,X,Y,IA,IB)
      DIMENSION X(IDM),Y(IDM),IA(IDM),IB(IDM)
      NX=A*5.+1.5
      NY=B*5.+1.5
```

```
      IF(NX.LT.4)NX=4
      IF(NY.LT.4)NY=4
      DX=A/NX
      DY=B/NY
      NM=2*(NX+NY)
      DO 3 J=1,NM
      IA(J)=J
      IB(J)=J+1
  3   IF (J .EQ. NM) IB(J)=1
      NP=NM
      NYP=NY+1
      IT=2*NY+NX+2
      AT=.5*A
      BT=.5*B
      DO 1 I=1,NYP
      II=IT-I
      X(I)=AT
      Y(I)=-BT+(I-1)*DY
      X(II)=-X(I)
  1   Y(II)=Y(I)
      IT=NYP+2*(NX-2)
      IS=NY+2
      IE=NX+NY
      DO 2 I=IS,IE
      II=NM-(I-IS)
      X(I)=AT-(I-IS+1)*DX
      Y(I)=BT
      X(II)=X(I)
 2    Y(II)=-Y(I)
      RETURN
      END
```

APPENDIX C
COMPUTER PROGRAMS FOR RADIATION AND SCATTERING
FROM TE DIELECTRIC-COATED CYLINDERS

```
      COMPLEX CQT,ZS,Z11,Z12,Y11,Y12,HZM,HZS,HZT
      COMPLEX C(56,56),CJ(56),HJJ(56),HMM(56),VJ(56),HJJEQ(56)
      DIMENSION IA(56),IB(56),I1(56),I2(56),I3(56),JA(56),JB(56)
      DIMENSION MD(56,5),ND(56),X(56),Y(56),D(56),XC(56),YC(56),DC(5
      COMMON/COA/TSK,ER2,CONST
      DATA IDM,INT/56,10/
      DATA PI,TP,ETA/3.14159,6.28318,376.727/
2     FORMAT(1X,8F15.7)
5     FORMAT(1H0)
7     FORMAT(7F10.5)
8     FORMAT(1X,14I5)
      CQT=1.414214*ETA*CMPLX(1.,-1.)
10    READ(5,8)IWR,LOP,NM,NP
      WRITE(6,8)IWR,LOP,NM,NP
      WRITE(6,5)
      DO 50 J=1,NM
      READ(5,8)IA(J),IB(J)
50    WRITE(6,8)J,IA(J),IB(J)
      WRITE(6,5)
      CALL SORT(IA,IB,I1,I2,I3,JA,JB,MD,ND,NM,NP,N,IDM,MAX,MIN)
      IF(MIN.LT.1 .OR. MAX.GT.5)GO TO 300
      DO 55 I=1,N
55    WRITE(6,8)I,I1(I),I2(I),I3(I)
      WRITE(6,5)
      DO 60 I=1,NP
      READ(5,7)XC(I),YC(I)
      FI=I
60    WRITE(6,2)FI,XC(I),YC(I)
      WRITE(6,5)
      DO 70 JAN=1,NM
      KIM=IA(JAN)
      LIM=IB(JAN)
70    DC(JAN)=SQRT((XC(LIM)-XC(KIM))**2+(YC(LIM)-YC(KIM))**2)
      READ(5,7) TSL,ER2
      WRITE(6,2) TSL,ER2
      WRITE(6,5)
      TSK=TP*TSL
      CONST=(ER2-1.)/ER2*(COS(TSK*SQRT(ER2-1.))-1.)
80    READ(5,7)CMM,DPH,FMC,SCALE,TC
      WRITE(6,2)CMM,DPH,FMC,SCALE,TC
      WRITE(6,5)
      WAVM=300./FMC
      TPL=TP
```


C8

```
      IF(SCALF.GT.0.)TPL=TP*SCALE/WAVM
      DO 90 IAN=1,NP
      X(IAN)=TPL*XC(IAN)
 90   Y(IAN)=TPL*YC(IAN)
      DO 95 JAN=1,NM
 95   D(JAN)=TPL*DC(JAN)
      I12=1
      ISYM=0
      ZS=(.0,.0)
      TK=TPL*TC
      IF(CMM.GT.0.)CALL CSURF(CMM,FMC,TK,ZS)
      IF(CMM.GT.0.)ISYM=1
      IF(TSL.GT.0.0) ISYM=1
      CALL CDANT(C,D,X,Y,ZS,IA,IB,I1,I2,I3,ISYM
     8,IDM,INT,JA,JB,MD,N,ND,NM,NP)
      IF(ISYM.EQ.10)GO TO 300
      GO TO (110,120,130,140),LOP
 110  READ(5,8)IGN
      CALL VNAS(IDM,IGN,ISYM,IWR,I12,N,C,CJ,Y11)
      FGN=IGN
      WRITE(6,2)FGN,Y11
      GO TO 200
 120  READ(5,8)JSA,JSB
      CALL VWAS(IA,IB,IDM,ISYM,IWR,I1,I2,I3,I12,JSA,JSB,MD,N,ND,NM
     2,C,CJ,D,VJ,Y11)
      FSA=JSA
      FSB=JSB
      WRITE(6,2)FSA,FSB,Y11
      GO TO 200
 130  READ(5,7)PSI,XCS,YCS
      XS=TPL*XCS
      YS=TPL*YCS
      CALL VMLS(IA,IB,IDM,INT,ISYM,IWR,I1,I2,I3,I12,MD,N,ND,NM,
     2C,CJ,D,PSI,VJ,X,Y,XS,YS,Y11,ZS)
      WRITE(6,2)PSI,XCS,YCS,Y11
      GO TO 200
 140  READ(5,7)BSC,PHI
      WRITE(6,2)BSC,PHI
 200  WRITE(6,5)
      IF(LOP.NE.4)G=REAL(Y11)
      INC=-1
      IF(LOP.EQ.4)INC=1
      IPA=2
      IF(LOP.EQ.4)IPA=1
      IF(LOP.NE.4)BSC=-1.
      NPH=360./DPH+1.5
      GR=.0
      DO 280 IPH=IPA,NPH
      FPH=IPH-2
      PH=DPH*FPH
```

```
      IF(IPH.EQ.1)PH=PHI
      CALL VFF(IA,IB,INC,IDM,ISYM,IWR,I1,I2,I3,I12,LOP,MD,N,ND,NM,
     2C,CJ,D,EWL,G,GAIN,HJJ,HMM,HZS,HZT,PH,ECS,VJ,X,Y,XS,YS,ZS)
      IF(LOP.NE.4)WRITE(6,2)PH,GAIN
      IF(LOP.EQ.4)WRITE(6,2)PH,EWL,ECS
      INC=-1
      IF(BSC.GT.0.)INC=1
  280 GR=GR+CABS(HZT)**2
      WRITE(6,5)
      SCS=.0174533*DPH*GR
      IF(LOP.EQ.4)WRITE(6,2)SCS
      GR=ETA*SCS
      IF(LOP.NE.4)WRITE(6,2)GR
      WRITE(6,5)
      READ(5,8)JOB,LOP
      IF(JOB.EQ.10)GO TO 10
      IF(JOB.EQ.80)GO TO 80
      IF(JOB.EQ.300)GO TO 300
      GO TO(110,120,130,140),LOP
  300 CONTINUE
      CALL EXIT
      END
      SUBROUTINE SORT(IA,IB,I1,I2,I3,JA,JB,MD,ND,NM,NP,N,MAX,MIN
     2,ICJ,INM)
      DIMENSION IA(1),IB(1),ND(1),MD(INM,4),JSP(20)
      DIMENSION I1(1),I2(1),I3(1),JA(1),JB(1)
    9 FORMAT(3X,'MAX = ',I5,3X,'MIN = ',I5,3X,'N = ',I5)
      I=0
      DO 24 K=1,NP
      NJK=0
      DO 20 J=1,NM
      IND=(IA(J)-K)*(IB(J)-K)
      IF(IND.NE.0)GO TO 20
      NJK=NJK+1
      JSP(NJK)=J
   20 CONTINUE
      MOD=NJK-1
      IF(MOD.LE.0)GO TO 24
      DO 22 IMD=1,MOD
      I=I+1
      IF(I.GT.ICJ)GO TO 22
      IPD=IMD+1
      JAI=JSP(IMD)
      JA(I)=JAI
      JBI=JSP(IPD)
      JB(I)=JBI
      I1(I)=IA(JAI)
      IF(IA(JAI).EQ.K)I1(I)=IB(JAI)
      I2(I)=K
      I3(I)=IA(JBI)
      IF(IA(JBI).EQ.K)I3(I)=IB(JBI)
   22 CONTINUE
```

```
24    CONTINUE
      N=I
      DO 30 J=1,NM
      ND(J)=0
      DO 30 K=1,4
30    MD(J,K)=0
      III=N
      IF(N.GT.ICJ)III=ICJ
      DO 40 I=1,III
      J=JA(I)
      DO 38 L=1,2
      ND(J)=ND(J)+1
      K=1
      M=0
32    MJK=MD(J,K)
      IF(MJK.NE.0)GO TO 34
      M=1
      MD(J,K)=I
34    K=K+1
      IF(K.GT.4)GO TO 38
      IF(M.EQ.0)GO TO 32
38    J=JB(I)
40    CONTINUE
      MIN=100
      MAX=0
      DO 46 J=1,NM
      NDJ=ND(J)
      IF(NDJ.GT.MAX)MAX=NDJ
46    IF(NDJ.LT.MIN)MIN=NDJ
      IF(MAX.GT.4 .OR. MIN.LT.1 .OR. N.GT.ICJ)WRITE(6,9)MAX,MIN,N
      RETURN
      END
      SUBROUTINE CDANT(C,D,X,Y,ZS,IA,IB,I1,I2,I3,ISYM
     2,IDM,INT,JA,JB,MD,N,ND,NM,NP)
      COMPLEX ZS,P11,P12,P21,P22,Q11,Q12,Q21,Q22,P(2,2),Q(2,2)
      COMPLEX C(IDM,IDM)
      DIMENSION X(1),Y(1),D(1),IA(1),IB(1),JA(1),JB(1)
      DIMENSION I1(1),I2(1),I3(1),MD(IDM,4),ND(1)
2     FORMAT(3X,'DMAX = ',E10.3,3X,'DMIN = ',E10.3)
      DO 20 I=1,N
      DO 20 J=1,N
20    C(I,J)=(.0,.0)
      DMAX=.0
      DMIN=100.
      DO 25 J=1,NM
      K=IA(J)
      L=IB(J)
      D(J)=SQRT((X(L)-X(K))**2+(Y(L)-Y(K))**2)
      IF(D(J).GT.DMAX)DMAX=D(J)
25    IF(D(J).LT.DMIN)DMIN=D(J)
      DRAT=DMIN/DMAX
      IF(DMAX.LT.3. .AND. DRAT.GT..01)GO TO 30
      N=0
      WRITE(6,2)DMAX,DMIN
```

```
      RETURN
   30 DO 200 K=1,NM
      NDK=ND(K)
      KA=IA(K)
      KB=IB(K)
      DK=D(K)
      DO 200 L=1,NM
      NDL=ND(L)
      LA=IA(L)
      LB=IB(L)
      DL=D(L)
      NIL=0
      DO 200 II=1,NDK
      I=MD(K,II)
      FI=1.
      IF(KB.EQ.I2(I))GO TO 36
      IF(KB.EQ.I1(I))FI=-1.
      IS=1
      GO TO 40
   36 IF(KA.EQ.I3(I))FI=-1.
      IS=2
   40 DO 200 JJ=1,NDL
      J=MD(L,JJ)
      IF(ISYM.NE.0)GO TO 42
      IF(I.GT.J)GO TO 200
   42 FJ=1.
      IF(LB.EQ.I2(J))GO TO 46
      IF(LB.EQ.I1(J))FJ=-1.
      JS=1
      GO TO 50
   46 IF(LA.EQ.I3(J))FJ=-1.
      JS=2
   50 IF(NIL.NE.0)GO TO 168
      NIL=1
      IF(K.EQ.L)GO TO 120
      IND=(LA-KA)*(LB-KA)*(LA-KB)*(LB-KB)
      IF(IND.EQ.0)GO TO 80
C     SEGMENTS K AND L SHARE NO POINTS
      CALL ZMM3(X(KA),Y(KA),X(KB),Y(KB),X(LA),Y(LA),X(LB),Y(LB),ZS,
     2DK,DL,INT,P(1,1),P(1,2),P(2,1),P(2,2))
      GO TO 168
C     SEGMENTS K AND L SHARE ONE POINT (THEY INTERSECT)
   80 KG=3
      JM=KB
      JC=KA
      KF=-1
      IND=(KB-LA)*(KB-LB)
      IF(IND.NE.0)GO TO 82
      JC=KB
```

```
      KF=1
      JM=KA
      KG=0
   82 LG=3
      JP=LA
      LF=-1
      IF(LB.EQ.JC)GO TO 83
      JP=LB
      LF=1
      LG=0
   83 SGN=KF*LF
      CALL ZMM2(X(JM),Y(JM),X(JC),Y(JC),X(JP),Y(JP),ZS,DK,DL,
     2INT,Q(1,1),Q(1,2),Q(2,1),Q(2,2))
      DO 98 KK=1,2
      KP=IABS(KK-KG)
      DO 98 LL=1,2
      LP=IABS(LL-LG)
   98 P(KP,LP)=SGN*Q(KK,LL)
      GO TO 168
C     K=L  (SELF REACTION OF SEGMENT K)
  120 CALL ZMM1(DK,ZS,P(1,1),P(1,2))
      P(2,1)=P(1,2)
      P(2,2)=P(1,1)
  168 C(I,J)=C(I,J)+FI*FJ*P(IS,JS)
  200 CONTINUE
      RETURN
      END
      SUBROUTINE ZMM1(DK,ZS,P11,P12)
      COMPLEX ZS,H0,H1,P11,P12
C
      COMMON/COA/TSK,ER2,CONST
C
      DATA PI/3.14159/
      CDK=COS(DK)
      SDK=SIN(DK)
      CALL HANK(DK,H0,H1,2)
      SDKS=SDK**2
      CDKS=CDK**2
      P11 =-2.*H1*CDK+H0*SDK+2.*(.0,1.)*(1.+CDKS)/PI/DK
      P12 =-H0*CDK*SDK+H1*(1.+CDKS)-4.*(.0,1.)*CDK/PI/DK
      P11=15.*DK*P11/SDKS*(1.+CONST)
      P12=15.*DK*P12/SDKS*(1.+CONST)
C
      TDK=2.*DK
      CTDK=COS(TDK)
      STDK=SIN(TDK)
      CCT=SIN(TSK*SQRT(ER2-1.))/SQRT(ER2-1.)
      P11=P11+CCT*(ER2-1.)/ER2*(.0,7.5)*(TDK+STDK)/SDKS
      P12=P12-(CCT*(ER2-1.)/ER2*(.0,7.5))*
     2(CDK*(TDK+STDK)+SDK*(1.-CTDK))/SDKS
```

```
C
      RS=CABS(ZS)
      IF(RS.LE.0.)GO TO 100
      CST=16.*PI*SDKS
      TDK=2.*DK
      CTDK=COS(TDK)
      STDK=SIN(TDK)
      P11=P11+ZS*(TDK-STDK)/CST
      P12 =P12 +ZS*((1.-CTDK)*SDK+(STDK-TDK)*CDK)/CST
  100 RETURN
      END
      SUBROUTINE ZMM2(X1,Y1,X2,Y2,X3,Y3,ZS
     2,DK1,DK2,INT,Q11,Q12,Q21,Q22)
       COMPLEX H0,H1,HH0,HH1,SH0,SH1,Q11,Q12,Q21,Q22
       COMPLEX DHH0,DHH1,DH0,DH1,DSH0,DSH1
       COMPLEX S11,S12,S21,S22,T11,T12,T21,T22,Y11,Y12,Y21,Y22
       COMPLEX DT11,DT12,DT21,DT22,DY11,DY12,DY21,DY22
       COMPLEX ZS,RKH1,SX1,SX2,CCP,FUN,CQT
C
       COMPLEX CX1,CX2,DS11,DS12,DS21,DS22
       COMPLEX DP11,DP12,DP21,DP22,P11,P12,P21,P22
       COMMON/COA/TSK,ER2,CONST
C
       DATA CCP,PI/(.0,.63662),3.14159/
       SDK1=SIN(DK1)
       SDK2=SIN(DK2)
       CDK1=COS(DK1)
       CDK2=COS(DK2)
       CBET=(X2-X1)/DK1
       SBET=(Y2-Y1)/DK1
       XB=(X3-X1)*CBET+(Y3-Y1)*SBET
       YB=-(X3-X1)*SBET+(Y3-Y1)*CBET
       CAL=(XB-DK1)/DK2
       SAL=ABS(YB/DK2)
       CALL HANK(DK2,HH0,HH1,2)
       DHH0=DK2*HH0
       DHH1=DK2*HH1
       C1S2=CDK1*SDK2
       C1C2=CDK1*CDK2
       IF(CAL.LT.0.)GO TO 20
       IF(SAL.GT..04)GO TO 20
       CNT=-15.*CAL/SDK1/SDK2
       CALL HANK(DK1,H0,H1,2)
       DH0=DK1*H0
       DH1=DK1*H1
       DKS=DK1+DK2
       CALL HANK(DKS,SH0,SH1,2)
       DSH0=DKS*SH0
       DSH1=DKS*SH1
       Q11=CNT*(CDK1*DSH1-C1S2*DH0-C1C2*DH1-DHH1+CCP*CDK2)
       Q12=CNT*(CDK2*DHH1-SDK2*DHH0-CCP+CDK1*DH1+C1S2*DSH0-C1C2*DSH1)
       Q21=CNT*(SDK2*DH0-DSH1+CDK2*DH1+CDK1*DHH1-CCP*C1C2)
       Q22=CNT*(C1S2*DHH0-C1C2*DHH1+CCP*CDK1-DH1-SDK2*DSH0+CDK2*DSH1)
```

```
      Q11=Q11*(1.+CONST)
      Q12=Q12*(1.+CONST)
      Q21=Q21*(1.+CONST)
      Q22=Q22*(1.+CONST)
      RETURN
   20 S11=-DHH1+CCP*CDK2
      S12=-SDK2*DHH0+CDK2*DHH1-CCP
      S21=(DHH1-CCP*CDK2)*CDK1
      S22=(SDK2*DHH0-CDK2*DHH1+CCP)*CDK1
C
      DS11=DHH0+CCP*SDK2
      DS12=-CDK2*DHH0-SDK2*DHH1
      DS21=CDK1*(-DS11)
      DS22=CDK1*(-DS12)
C
      DKS1=DK1**2
      AL=ATAN2(SAL,CAL)
      RMIN=DK1
      IF(CAL.GE.0.)GO TO 30
      RMIN=DK1*SAL
      DCR=-DK1*CAL
      IF(DK2.LT.DCR)RMIN=SQRT(DKS1+2.*DK1*DK2*CAL+DK2*DK2)
   30 FNT=1+(4*INT)/10
      INP=FNT*DK2/RMIN
      INP=2*(INP/2)
      IF(INP.LT.2)INP=2
      IP=INP+1
      DT=DK2/INP
      TK=.0
      SX1=(.0,.0)
      SX2=(.0,.0)
C
      CX1=(.0,.0)
      CX2=(.0,.0)
C
      SGI=-1.
      DO 90 I=1,IP
      D=SGI+3.
      IF(I.EQ.1 .OR. I.EQ.IP)D=1.
      TKS=TK*TK
      RK=SQRT(DKS1+2.*DK1*TK*CAL+TKS)
      CALL HANK(RK,H0,H1,0)
      S1=SIN(DK2-TK)
      S2=SIN(TK)
      SX1=SX1+S1*H0*D
      SX2=SX2+S2*H0*D
C
      C1=-COS(DK2-TK)
      C2=COS(TK)
      CX1=CX1+C1*H0*D
      CX2=CX2+C2*H0*D
```

```
C
      SGI=-SGI
   90 TK=TK+DT
      SX1=SX1*DT/3.
      SX2=SX2*DT/3.
      S21=S21-SX1
      S22=S22-SX2
      S12=S12+CDK1*SX2
      S11=S11+CDK1*SX1
C
      CX1=CX1*DT/3.
      CX2=CX2*DT/3.
      DS11=DS11+CDK1*CX1
      DS12=DS12+CDK1*CX2
      DS21=DS21-CX1
      DS22=DS22-CX2
C
      INP=2*(INT/2)
      IP=INP+1
      JP=IP
      T11=(.0,.0)
      T12=(.0,.0)
      T21=(.0,.0)
      T22=(.0,.0)
      COT=(.0,.0)
      RS=CABS(ZS)
      Y11=(.0,.0)
      Y12=(.0,.0)
      Y21=(.0,.0)
      Y22=(.0,.0)
C
      P11=(.0,.0)
      P12=(.0,.0)
      P21=(.0,.0)
      P22=(.0,.0)
C
      B=.0
      IF(AL.LT..05)GO TO 210
      ALT=AL/2.
      CALT=COS(ALT)
      SALT=SIN(ALT)
      RCP=(DK1+DK2)*CALT
      RSP=(DK2-DK1)*SALT
      PHC=ATAN2(RSP,RCP)
      SGI=-1.
      PH=-ALT
      DPH=AL/INP
      DO 200 I=1,IP
      D=SGI+3.
      IF(I.EQ.1 .OR. I.EQ.IP)D=1.
      SAP=SIN(ALT+PH)
      SAM=SIN(ALT-PH)
```

```
      IF(PH.LE.PHC)RMAX=DK1*SAL/SAM
      IF(PH.GT.PHC)RMAX=DK2*SAL/SAP
      DRK=RMAX/INP
      RK=.0
      SGJ=-1.
      DT11=(.0,.0)
      DT12=(.0,.0)
      DT21=(.0,.0)
      DT22=(.0,.0)
      DY11=(.0,.0)
      DY12=(.0,.0)
      DY21=(.0,.0)
      DY22=(.0,.0)
C
      DP11=(.0,.0)
      DP12=(.0,.0)
      DP21=(.0,.0)
      DP22=(.0,.0)
C
      RKH1=CCP
      DO 100  J=1,JP
      C=SGJ+3.
      IF(J.EQ.1 .OR. J.EQ.JP)C=1.
      IF(J.EQ.1)GO TO 94
      CALL HANK(RK,H0,H1,1)
      RKH1=RK*H1
   94 CONTINUE
      SK=RK*SAM/SAL
      TK=RK*SAP/SAL
      C1=COS(SK)
      C2=COS(DK1-SK)
      S1=SIN(DK2-TK)
      S2=SIN(TK)
      FUN=C*RKH1
      DY11=DY11-FUN*C1*S1
      DY12=DY12-FUN*C1*S2
      DY21=DY21+FUN*C2*S1
      DY22=DY22+FUN*C2*S2
C
      S1P=-COS(DK2-TK)
      S2P=COS(TK)
      DP11=DP11-FUN*S1P*C1
      DP12=DP12-FUN*S2P*C1
      DP21=DP21+FUN*S1P*C2
      DP22=DP22+FUN*S2P*C2
C
      SGJ=-SGJ
      RK=RK+DRK
      IF(RS.LE.0.)GO TO 100
      SS1=SIN(SK)
      SS2=SIN(DK1-SK)
```

```
      DT11=DT11+FUN*SS1*S1
      DT12=DT12+FUN*SS1*S2
      DT21=DT21+FUN*SS2*S1
      DT22=DT22+FUN*SS2*S2
  100 CONTINUE
      B=SAP*DRK*D
      Y11=Y11+B*DY11
      Y12=Y12+B*DY12
      Y21=Y21+B*DY21
      Y22=Y22+B*DY22
C
      P11=P11+B*DP11
      P12=P12+B*DP12
      P21=P21+B*DP21
      P22=P22+B*DP22
C
      PH=PH+DPH
      SGI=-SGI
      IF(RS.LE.0.)GO TO 200
      T11=T11+B*DT11
      T12=T12+B*DT12
      T21=T21+B*DT21
      T22=T22+B*DT22
  200 CONTINUE
      B=DPH/9.
      IF(RS.GT.0.)CQT=(.0,1.)*ZS*DPH/(72.*PI*SDK1*SDK2*SAL)
  210 CNT=-15./SDK1/SDK2
      Q11=CNT*(CAL*(1.+CONST)*S11+B*Y11)+CQT*T11
      Q12=CNT*(CAL*(1.+CONST)*S12+B*Y12)+CQT*T12
      Q21=CNT*(CAL*(1.+CONST)*S21+B*Y21)+CQT*T21
      Q22=CNT*(CAL*(1.+CONST)*S22+B*Y22)+CQT*T22
C
      CCT=SIN(TSK*SQRT(ER2-1.))/SQRT(ER2-I.)
      Q11=Q11-CNT*CCT*(ER2-1.)/ER2*(DS11*SAL-B*P11*CAL/SAL)
      Q12=Q12-CNT*CCT*(ER2-1.)/ER2*(DS12*SAL-B*P12*CAL/SAL)
      Q21=Q21-CNT*CCT*(ER2-1.)/ER2*(DS21*SAL-B*P21*CAL/SAL)
      Q22=Q22-CNT*CCT*(ER2-1.)/ER2*(DS22*SAL-B*P22*CAL/SAL)
C
      RETURN
      END
      SUBROUTINE ZMM3(X1,Y1,X2,Y2,X3,Y3,X4,Y4,ZS,
     2DK1,DK2,INT,P11,P12,P21,P22)
      COMPLEX HHA,HHB,ZS,HZ1,HZ2,CQT,ET1,ET2,H0,H1
      COMPLEX P11,P12,P21,P22,S11,S12,S21,S22,T11,T12,T21,T22
C
      COMPLEX EQ1,EQ2,Q11,Q12,Q21,Q22
      COMMON/COA/TSK,ER2,CONST
C
      DIMENSION CC1(21),SS1(21),CC2(21),SS2(21)
      DATA ETA,PI/376.727,3.14159/
      S11=(.0,.0)
      S12=(.0,.0)
      S21=(.0,.0)
```

```
      S22=(.0,.0)
      T11=(.0,.0)
      T12=(.0,.0)
      T21=(.0,.0)
      T22=(.0,.0)
C
      Q11=(.0,.0)
      Q12=(.0,.0)
      Q21=(.0,.0)
      Q22=(.0,.0)
C
      CBET=(X2-X1)/DK1
      SBET=(Y2-Y1)/DK1
      XA=(X3-X1)*CBET+(Y3-Y1)*SBET
      XB=(X4-X1)*CBET+(Y4-Y1)*SBET
      YA=-(X3-X1)*SBET+(Y3-Y1)*CBET
      YB=-(X4-X1)*SBET+(Y4-Y1)*CBET
      CAL=(XB-XA)/DK2
      SAL=(YB-YA)/DK2
      RMIN=10000.
      X=XA
      Y=YA
      DX=DK2*CAL/4.
      DY=DK2*SAL/4.
      DO 40 J=1,5
      YS=.0
      R=ABS(Y)
      IF(R.GT.1.E-15)YS=Y*Y
      XS=.0
      XAB=ABS(X-DK1)
      IF(XAB.GT.1.E-15)XS=XAB*XAB
      IF(X.LT.0.)R=SQRT(X*X+YS)
      IF(X.GT.DK1)R=SQRT(XS+YS)
      IF(R.LT.RMIN)RMIN=R
      X=X+DX
   40 Y=Y+DY
      FNT=1+(4*INT)/10
      ISS=FNT*DK1/RMIN
      ISS=2*(ISS/2)
      IF(ISS.LT.2)ISS=2
      IF(ISS.GT.20)ISS=20
      FSS=ISS
      ISQ=ISS+1
      DS=DK1/FSS
      ITT=FNT*DK2/RMIN
      ITT=2*(ITT/2)
      IF(ITT.LT.2)ITT=2
      IF(ITT.GT.20)ITT=20
      FTT=ITT
      ITQ=ITT+1
      DT=DK2/FTT
      XP=.0
```

```
      SGN=-1.
      RS=CABS(ZS)
      JUMP=0
      ASAL=ABS(SAL)
      IF(RS.LE.0..AND.ASAL.LT..04)JUMP=1
      IF(JUMP.EQ.1)GO TO 60
      DO 50 I=1,ISQ
      C=SGN+3.
      IF(I.EQ.1 .OR. I.EQ.ISQ)C=1.
      CC1(I)=C*COS(DK1-XP)
      SS1(I)=C*SIN(DK1-XP)
      CC2(I)=C*COS(XP)
      SS2(I)=C*SIN(XP)
      SGN=-SGN
   50 XP=XP+DS
   60 DX=DT*CAL
      DY=DT*SAL
      X=XA
      Y=YA
      TK=.0
      SGJ=-1.
      CDK1=COS(DK1)
      DO 200 J=1,ITQ
      D=SGJ+3.
      IF(J.EQ.1 .OR. J.EQ.ITQ)D=1.
      CT1=D*SIN(DK2-TK)
      CT2=D*SIN(TK)
C
      CTP1=-D*COS(DK2-TK)
      CTP2=D*COS(TK)
C
      XP=.0
      YS=.0
      YAB=ABS(Y)
      IF(YAB.GT.1.E-15)YS=YAB*YAB
      ET1=(.0,.0)
      ET2=(.0,.0)
      HZ1=(.0,.0)
      HZ2=(.0,.0)
      RKA=SQRT(X*X+YS)
      RKB=SQRT((X-DK1)**2+YS)
      SPH=YAB/RKA+YAB/RKB
      IF(SPH.LT..04 .OR. JUMP.EQ.1)GO TO 110
      DO 100 I=1,ISQ
      DELX=ABS(X-XP)
      DXS=.0
      IF(DELX.GT.1.E-15)DXS=DELX*DELX
      RK=SQRT(DXS+YS)
      SPH=Y/RK
      C1=CC1(I)
      S1=SS1(I)
      C2=CC2(I)
      S2=SS2(I)
```

```
      CALL HANK(RK,HO,H1,1)
      ET1=ET1-C1*SPH*H1
      ET2=ET2+C2*SPH*H1
      XP=XP+DS
      IF(RS.LE.O.)GO TO 100
      HZ1=HZ1+S1*H1*SPH
      HZ2=HZ2+S2*H1*SPH
  100 CONTINUE
  110 CALL HANK(RKA,HHA,H1,0)
      CALL HANK(RKB,HO,H1,0)
C
      EQ1=ET1*CAL*DS/3.-SAL*(CDK1*HHA-HO)
      EQ2=ET2*CAL*DS/3.-SAL*(CDK1*HO-HHA)
      Q11=Q11+CTP1*EQ1
      Q12=Q12+CTP2*EQ1
      Q21=Q21+CTP1*EQ2
      Q22=Q22+CTP2*EQ2
C
      ET1=ET1*SAL*DS/3.+CAL*(CDK1*HHA-HO)
      ET2=ET2*SAL*DS/3.+CAL*(CDK1*HO-HHA)
      S11=S11+CT1*ET1
      S12=S12+CT2*ET1
      S21=S21+CT1*ET2
      S22=S22+CT2*ET2
      SGJ=-SGJ
      TK=TK+DT
      X=X+DX
      Y=Y+DY
      IF(RS.LE.O.)GO TO 200
      T11=T11+CT1*HZ1
      T12=T12+CT2*HZ1
      T21=T21+CT1*HZ2
      T22=T22+CT2*HZ2
  200 CONTINUE
      SDK1=SIN(DK1)
      SDK2=SIN(DK2)
      CST=-ETA*DT/(24.*PI*SDK1*SDK2)
      CQT=(.0,1.)*DS*DT*ZS/(72.*PI*SDK1*SDK2)
      P11=CST*(1.+CONST)*S11+CQT*T11
      P12=CST*(1.+CONST)*S12+CQT*T12
      P21=CST*(1.+CONST)*S21+CQT*T21
      P22=CST*(1.+CONST)*S22+CQT*T22
C
CC
      CCT=SIN(TSK*SQRT(ER2-1.))/SQRT(ER2-1.)
      CCT=CCT*(ER2-1.)/ER2
      P11=P11-CCT*Q11*CST
      P12=P12-CCT*Q12*CST
      P21=P21-CCT*Q21*CST
      P22=P22-CCT*Q22*CST
C
```

```
      RETURN
      END
      SUBROUTINE HANK(X,H,H1,ID)
      COMPLEX H,H1
      DATA TSP/.63661977/
      IF(X.GT.3.)GO TO 100
      XLN=TSP*ALOG(X/2.)
      B=.0
      B1=.0
      Y=.0
      Y1=.0
      X1=X/3.
      X2=X1*X1
      IF(X1.LT..1)GO TO 60
      X4=X2*X2
      X6=X2*X4
      IF(X1.LT..3)GO TO 55
      X8=X2*X6
      X10=X2*X8
      X12=X2*X10
      B=.21E-3*X12-.39444E-2*X10+.444479E-1*X8
      Y=-.24846E-3*X12+.427916E-2*X10-.4261214E-1*X8
      B1=.1109E-4*X12-.31761E-3*X10+.443319E-2*X8
      Y1=.27873E-2*X12-.400976E-1*X10+.3123951*X8
   55 B=B-.3163866*X6+1.2656208*X4
      Y=Y+.25300117*X6-.74350384*X4
      B1=B1-.3954289E-1*X6+.21093573*X4
      Y1=Y1-1.3164827*X6+2.1682709*X4
   60 B=B-2.2499997*X2+1.
      Y=Y+.60559366*X2+.36746691+XLN*B
      B1=X*(B1-.56249985*X2+.5)
      Y1=(Y1+.2212091*X2-.6366198)/X+XLN*B1
      GO TO 200
  100 SW=SQRT(X)
      X1=3./X
      X2=X1*X1
      X3=X1*X2
      X4=X1*X3
      X5=X1*X4
      X6=X1*X5
      F=.79788456-.77E-6*X1-.55274E-2*X2-.9512E-4*X3+.137237E-2*X4
     2-.72805E-3*X5+.14476E-3*X6
      T=X-.78539816-.4166397E-1*X1-.3954E-4*X2+.262573E-2*X3
     2-.54125E-3*X4-.29333E-3*X5+.13558E-3*X6
      B=F*COS(T)/SW
      Y=F*SIN(T)/SW
      F=.79788456+.156E-5*X1+.1659667E-1*X2+.17105E-3*X3-.249511E-2*X4
     2+.113653E-2*X5-.20033E-3*X6
      T=X-2.3561945+.12499612*X1+.565E-4*X2-.637879E-2*X3+.74348E-3*X4
     2+.79824E-3*X5-.29166E-3*X6
      B1=F*COS(T)/SW
      Y1=F*SIN(T)/SW
```

```
200 H=CMPLX(B,-Y)
    H1=CMPLX(B1,-Y1)
    RETURN
    END
    SUBROUTINE CROUT(C,S,ICC,ISYM,IWR,I12,N)
    COMPLEX C(ICC,ICC),S(1)
    COMPLEX F,P,SS,T
2   FORMAT(1X,1I5,1F10.3,1F15.7,1F10.0)
5   FORMAT(1H0)
    IF(I12.NE.1)GO TO 22
    IF(N.EQ.1)S(1)=S(1)/C(1,1)
    IF(N.EQ.1)GO TO 100
    IF(ISYM.NE.0)GO TO 8
    DO 6 I=1,N
    DO 6 J=I,N
6   C(J,I)=C(I,J)
8   F=C(1,1)
    DO 10 L=2,N
10  C(1,L)=C(1,L)/F
    DO 20 L=2,N
    LLL=L-1
    DO 20 I=L,N
    F=C(I,L)
    DO 11 K=1,LLL
11  F=F-C(I,K)*C(K,L)
    C(I,L)=F
    IF(L.EQ.I)GO TO 20
    P=C(L,L)
    IF(ISYM.EQ.0)GO TO 15
    F=C(L,I)
    DO 12 K=1,LLL
12  F=F-C(L,K)*C(K,I)
    C(L,I)=F/P
    GO TO 20
15  F=C(I,L)
    C(L,I)=F/P
20  CONTINUE
22  DO 30 L=1,N
    P=C(L,L)
    T=S(L)
    IF(L.EQ.1)GO TO 30
    LLL=L-1
    DO 25 K=1,LLL
25  T=T-C(L,K)*S(K)
30  S(L)=T/P
    DO 38 L=2,N
    I=N-L+1
    II=I+1
    T=S(I)
    DO 35 K=II,N
35  T=T-C(I,K)*S(K)
```

```
38   S(I)=T
     IF(IWR.LE.0) GO TO 100
     CNOR=.0
     DO 40 I=1,N
     SA=CABS(S(I))
40   IF(SA.GT.CNOR)CNOR=SA
     IF(CNOR.LE.0.)CNOR=1.
     DO 44 I=1,N
     SS=S(I)
     SA=CABS(SS)
     SNOR=SA/CNOR
     PH=.0
     IF(SA.GT.0.)PH=57.29578*ATAN2(AIMAG(SS),REAL(SS))
44   WRITE(6,2)I,SNOR,SA,PH
     WRITE(6,5)
100  RETURN
     END
     SUBROUTINE VNAS(IDM,IGN,ISYM,IWR,I12,N,C,CJ,Y11)
     COMPLEX C(IDM,IDM),CJ(1),Y11
     DO 20 I=1,N
20   CJ(I)=(.0,.0)
     CJ(IGN)=(1.,0.)
     CALL CROUT(C,CJ,IDM,ISYM,IWR,I12,N)
     I12=2
     Y11=CJ(IGN)
     RETURN
     END
     SUBROUTINE VWAS(IA,IB,IDM,ISYM,IWR,I1,I2,I3,I12,JSA,JSB,MD,N,ND
    2,NM,C,CJ,D,VJ,Y11)
     COMPLEX C(IDM,IDM),CJ(1),VJ(1),Y11
     DIMENSION IA(1),IB(1),I1(1),I2(1),I3(1),MD(IDM,4),ND(1),D(1)
     AK=.0
     DO 20 K=JSA,JSB
20   AK=AK+D(K)
     DO 30 I=1,N
30   VJ(I)=(.0,.0)
     IF(JSB.GT.JSA)GO TO 200
     K=JSA
     DK=D(K)
     V=(1.-COS(DK))/(AK*SIN(DK))
     KA=IA(K)
     KB=IB(K)
     NDK=ND(K)
     DO 140 II=1,NDK
     I=MD(K,II)
     FI=1.
     IF(KB.EQ.I2(I))GO TO 136
     IF(KB.EQ.I1(I))FI=-1.
     GO TO 140
136  IF(KA.EQ.I3(I))FI=-1.
140  VJ(I)=VJ(I)+FI*V
```

```
      GO TO 280
  200 KA=IA(JSA)
      KB=IB(JSA)
      LA=IA(JSA+1)
      LB=IB(JSA+1)
      IND=(LA-KB)*(LB-KB)
      IF(IND.EQ.0)GO TO 210
      KA=IB(JSA)
      KB=IA(JSA)
  210 DO 250 K=JSA,JSB
      DK=D(K)
      V=(1.-COS(DK))/(AK*SIN(DK))
      NDK=ND(K)
      DO 240 II=1,NDK
      I=MD(K,II)
      FI=1.
      IF(KB.EQ.I2(I))GO TO 236
      IF(KB.EQ.I1(I))FI=-1.
      GO TO 240
  236 IF(KA.EQ.I3(I))FI=-1.
  240 VJ(I)=VJ(I)+FI*V
      IF(K.EQ.JSB)GO TO 250
      LA=IA(K+1)
      LB=IB(K+1)
      KA=KB
      KB=LA
      IF(LA.EQ.KA)KB=LB
  250 CONTINUE
  280 DO 300 I=1,N
  300 CJ(I)=VJ(I)
      CALL CROUT(C,CJ,IDM,ISYM,IWR,I12,N)
      I12=2
      Y11=(.0,.0)
      DO 400 I=1,N
  400 Y11=Y11+VJ(I)*CJ(I)
      RETURN
      END
      SUBROUTINE VMLS(IA,IB,IDM,INT,ISYM,IWR,I1,I2,I3,I12,MD,N,ND,NM,
     2C,CJ,D,PSI,VJ,X,Y,XS,YS,Y11,ZS)
      COMPLEX C(IDM,IDM),CJ(1),VJ(1),Y11,P1,P2,Q1,Q2,ZS
      DIMENSION IA(1),IB(1),I1(1),I2(1),I3(1)
      DIMENSION MD(IDM,4),ND(1),X(1),Y(1),D(1)
      DATA ETA,TP/376.727,6.28318/
      DO 100 I=1,N
      VJ(I)=(.0,.0)
  100 CJ(I)=(.0,.0)
      DO 240 K=1,NM
      KA=IA(K)
      KB=IB(K)
      CALL CMLS(PSI,X(KA),Y(KA),X(KB),Y(KB),XS,YS,D(K),INT,P1,P2,Q1,Q2
      Q1=ZS*Q1
      Q2=ZS*Q2
      NDK=ND(K)
```

```
      DO 240 II=1,NDK
      I=MD(K,II)
      FI=1.
      IF(KB.EQ.I2(I)) GO TO 236
      IF(KB.EQ.I1(I)) FI=-1.
      CJ(I)=CJ(I)+FI*P1
      VJ(I)=VJ(I)+FI*(P1+Q1)
      GO TO 240
  236 IF(KA.EQ.I3(I)) FI=-1.
      CJ(I)=CJ(I)+FI*P2
      VJ(I)=VJ(I)+FI*(P2+Q2)
  240 CONTINUE
      CALL CROUT(C,CJ,IDM,ISYM,IWR,I12,N)
      I12=2
      Y11=CMPLX(TP/(4.*ETA),0.)
      DO 300 I=1,N
  300 Y11=Y11+CJ(I)*VJ(I)
      RETURN
      END
      SUBROUTINE CMLS(PSI,X1,Y1,X2,Y2,XS,YS,DK,INT,P1,P2,Q1,Q2)
      COMPLEX CST,H0,H1,P1,P2,Q1,Q2
      DATA ETA/376.727/
      DKH=DK/25.
      D1=SQRT((XS-X1)**2+(YS-Y1)**2)
      D2=SQRT((XS-X2)**2+(YS-Y2)**2)
      P2=(.0,.0)
      P1=CMPLX(.5*PSI/360.,0.)
      Q1=(.0,.0)
      Q2=(.0,.0)
      IF(D1.LT.DKH)GO TO 200
      P1=(.0,.0)
      P2=CMPLX(.5*PSI/360.,0.)
      IF(D2.LT.DKH)GO TO 200
      SDK=SIN(DK)
      P1=(.0,.0)
      P2=(.0,.0)
      CBET=(X2-X1)/DK
      SBET=(Y2-Y1)/DK
      XA=(XS-X1)*CBET+(YS-Y1)*SBET
      YA=-(XS-X1)*SBET+(YS-Y1)*CBET
      X=XA
      Y=YA
      YSQ=Y**2
      RMIN=ABS(Y)
      IF(X.LT.0.)RMIN=SQRT(X*X+YSQ)
      IF(X.GT.DK)RMIN=SQRT((X-DK)**2+YSQ)
      FNT=1+(4*INT)/10
      ISS=FNT*DK/RMIN
      IF(ISS.LT.2)ISS=2
      DS=DK/ISS
      XP=DS/2.
```

```
      DO 100 I=1,ISS
      DELX=X-XP
      RK=SQRT(DELX**2+YSQ)
      SPH=Y/RK
      S1=SIN(DK-XP)
      S2=SIN(XP)
      CALL HANK(RK,H0,H1,2)
      P1=P1+S1*H1*SPH
      P2=P2+S2*H1*SPH
      Q1=Q1+S1*H0
      Q2=Q2+S2*H0
  100 XP=XP+DS
      CST=(.0,1.)*DS/(4.*SDK)
      P1=CST*P1
      P2=CST*P2
      CNT=-DS/(4.*ETA*SDK)
      Q1=CNT*Q1
      Q2=CNT*Q2
  200 RETURN
      END

      SUBROUTINE VFF(IA,IB,INC,IDM,ISYM,IWR,I1,I2,I3,I12,LOP,MD,N,ND,NM
     2C,CJ,D,EWL,C,GAIN,HJJ,HMM,HZS,HZT,PH,ECS,VJ,X,Y,XS,YS,ZS)
      COMPLEX CQT,DQT,HJ1,HJ2,HM1,HM2,HZM,HZS,HZT,ZS
      COMPLEX CJ(1),HJJ(1),HMM(1),C(IDM,IDM),VJ(1)
      COMPLEX HJ1J,HJ2J,HJJEQ(40)
      DIMENSION IA(1),IB(1),I1(1),I2(1),I3(1),MD(IDM,4)
      DIMENSION ND(1),X(1),Y(1),D(1)
      COMPLEX CONPH
      COMMON/COB/CONPH
      COMMON/COA/TSK,ER2,CONST
      DATA ETA,TP/376.727,6.28318/
      CQT=1.414214*ETA*CMPLX(1.,-1.)
      IF(ISYM.NE.0)DQT=CQT*CONJG(ZS)/ZS
      ECS=.0
      PHR=.0174533*PH
      CPH=COS(PHR)
      SPH=SIN(PHR)
      DO 232 I=1,N
      HJJEQ(I)=(.0,.0)
      HJJ(I)=(.0,.0)
  232 HMM(I)=(.0,.0)
      DO 250 K=1,NM
      KA=IA(K)
      KB=IB(K)
      CALL CFF(X(KA),Y(KA),X(KB),Y(KB),D(K)
     2,CPH,SPH,ZS,HJ1,HJ2,HM1,HM2,HJ1J,HJ2J)
      NDK=ND(K)
      DO 250 II=1,NDK
      I=MD(K,II)
      FI=1.
```

```
      IF(KB.EQ.I2(I))GO TO 236
      IF(KB.EQ.I1(I))FI=-1.
      HJJ(I)=HJJ(I)+FI*HJ1
      HMM(I)=HMM(I)+FI*HM1
      HJJEQ(I)=HJJEQ(I)+FI*HJ1J
      GO TO 250
  236 IF(KA.EQ.I3(I))FI=-1.
      HJJ(I)=HJJ(I)+FI*HJ2
      HMM(I)=HMM(I)+FI*HM2
      HJJEQ(I)=HJJEQ(I)+FI*HJ2J
  250 CONTINUE
      IF(INC.LE.0)GO TO 270
      DO 260 I=1,N
      CJ(I)=CQT*HJJ(I)
      VJ(I)=CJ(I)
  260 IF(ISYM.NE.0)VJ(I)=VJ(I)-DQT*HMM(I)
      CALL CROUT(C,CJ,IDM,ISYM,IWR,I12,N)
      I12=2
      DO 265 I=1,N
  265 ECS=ECS+REAL(VJ(I)*CONJG(CJ(I)))
      ECS=ECS/ETA
  270 HZS=(.0,.0)
      DO 360 I=1,N
  360 HZS=HZS+CJ(I)*(HJJ(I)+HMM(I))+CJ(I)*(CONPH*HJJ(I)+HJJEQ(I))
      HAB=CABS(HZS)
      IF(LOP.EQ.4)EWL=TP*HAB*HAB
      HZT=HZS
      IF(LOP.NE.3)GO TO 400
      PSI=XS*CPH+YS*SPH
      HZM=CMPLX(COS(PSI),SIN(PSI))
      HZM=-(1.,1.)*HZM/(2.*1.414214*ETA)
      HZT=HZS+HZM
  400 HAB=CABS(HZT)
      IF(LOP.LT.4)GAIN=TP*ETA*HAB*HAB/G
      RETURN
      END
      SUBROUTINE CFF(XA,YA,XB,YB,DK,CPH,SPH,ZS,HJ1,HJ2,HM1,HM2,
     2HJ1J,HJ2J)
      COMPLEX EJA,EJB,CST,ZS,HJ1,HJ2,HM1,HM2
      COMPLEX E1,E2,CSTJ,PHASE,FB1,FB2,HJ1J,HJ2J
      COMPLEX CONPH
      COMMON/COB/CONPH
      COMMON/COA/TSK,ER2,CONST
      DATA ETA,PI/376.727,3.14159/
      CA=(XB-XA)/DK
      CB=(YB-YA)/DK
      G=CA*CPH+CB*SPH
      P=CB*CPH-CA*SPH
      GK=P**2
      A=XA*CPH+YA*SPH
      B=XB*CPH+YB*SPH
```

```
      EJA=CMPLX(COS(A),SIN(A))
      EJB=CMPLX(COS(B),SIN(B))
      SDK=SIN(DK)
      CDK=COS(DK)
      IF(GK.LT..001)GO TO 250
      CST=CMPLX(1.,1.)/(4.*PI*SDK*1.414214*GK)
      HM1=CST*(EJA*CMPLX(CDK,G*SDK)-EJB)
      HM2=CST*(EJB*CMPLX(CDK,-G*SDK)-EJA)
      GO TO 300
  250 CST=CMPLX(-1.,1.)/(8.*PI)*1.414214*SDK)
      IF(G.LT.0.)GO TO 280
      HM1=CST*(DK*EJB-SDK*EJA)
      HM2=CST*(SDK*EJB-DK*EJA)
      GO TO 300
  280 HM1=CST*(SDK*EJA-DK*EJB)
      HM2=CST*(DK*EJA-SDK*EJB)
  300 HJ1=P*HM1
      HJ2=P*HM2
      HM1=-ZS*HM1/ETA
      HM2=-ZS*HM2/ETA
CC
      D=TSK/2./PI
      E1=CMPLX(.5*D,.0)
      E2=CMPLX(-.5*D,.0)
      SP=2.*PI*(SQRT(ER2-1.)+P)
      SM=2.*PI*(SQRT(ER2-1.)-P)
      SSP=SIN(SP*D)
      CSP=COS(SP*D)
      SSM=SIN(SM*D)
      CSM=COS(SM*D)
      IF(SP.GT.0.001)E2=(.0,-.5)*(CMPLX(CSP,-SSP)-1.)/SP
      IF(SM.GT.0.001)E1=(.0,-.5)*(CMPLX(CSM,SSM)-1.)/SM
      PHASE=E1-E2
      CONPH=(.0,1.)*(ER2-1.)/ER2*(-E1-E2)
      IF(GK.GT.0.001)GO TO 500
      FB1=.5*(DK*EJA+SDK*EJB)*PHASE
      FB2=.5*(DK*EJB+SDK*EJA)*PHASE
      GO TO 400
 500  CONTINUE
      FB1=PHASE*(CMPLX(.0,G)*EJB+CMPLX(SDK,-G*CDK)*EJA)
      FB2=PHASE*(CMPLX(.0,G)*EJA-CMPLX(SDK,G*CDK)*EJB)
 400  CONTINUE
      CSTJ=CMPLX(1.,1.)/(4.*PI*SDK*1.414214)
      IF(GK.GT..001)
     2CSTJ=CMPLX(1.,1.)/(4.*PI*SDK*1.414214*GK)
      OO=1.
      HJ1J=OO*CSTJ*(ER2-1.)/ER2*G*(-FB1)
      HJ2J=OO*CSTJ*(ER2-1.)/ER2*G*(-FB2)
```

```
CC
      RETURN
      END
      SUBROUTINE CSURF(CMM,FMC,TK,ZS)
      COMPLEX ETA,R,ZS,ETBT
      DATA E,ETAO,TP,U/8.85433E-12,376.727,6.28318,12.5664E-7/
      ALPH=SQRT(TP*FMC*U*CMM/2.)*1.E6
      SQT=SQRT(TP*FMC*U/(2.*CMM))
      ETA=CMPLX(SQT,SQT)
      TAT=2.*TK*SQRT(CMM/(2.*E*TP*FMC))
      ZS=ETA
      IF(TAT.GT.60.)GO TO 100
      ETAT=EXP(-TAT)
      ETBT=CMPLX(COS(TAT),-SIN(TAT))
      R=ETAT*ETBT*(ETAO-ETA)/(ETAO+ETA)
      ZS=ETA*(1.+R)/(1.-R)
  100 RETURN
      END
```

APPENDIX D
COMPUTER PROGRAMS FOR RADIATION AND SCATTERING
FROM PERFECTLY-CONDUCTING PLATES

```
      DIMENSION C(31),NN(40),MM(40)
      COMPLEX Z(40,40),Z11,Z12,ZSS,ZIJ
      COMPLEX VA,VB,VC,VD,VTA,VTB,VPA,VPB,EST
      COMPLEX VT(40),VP(40),ET(40),EP(40)
      IDM=40
      PI=3.141592
      TP=2.*PI
      READ(5,98)ZKL,TK,NS,NMAX,NW
   98 FORMAT(2F10.4,3I10)
  333 CONTINUE
      ZL=ZKL/PI
      TK=TK*TP
      NZ=NW
      CALL SIMWC(NMAX,C)
      NP=NS-1
      NZS=NZ-1
      MODES=NW*NZS
      MODET=2*MODES
      AL=ZL*2./NZ
      HL=AL/NS
      DWL=ZL/NZ
      DDW=DWL/(NMAX-1)
      DO 11 K=1,NW
      DO 11 L=1,NZS
      Z12=(0.0,0.0)
      NI=K+(L-1)*NW
      DO 30 II=1,1
      Y1=DDW*(II-1)*TP
      Y2=Y1
      Y3=Y1
      DO 30 JJ=1,NMAX
      DT=ATAN2(DWL,1.414)/(NMAX-1)
      CC=C(JJ)
      TH=DT*(JJ-1)
      DV=1.414*DWL/(NMAX-1)
      V=DV*(JJ-1)+(K-2)*0.707*DWL
      YA=V*TP*1.414
      IF(K.EQ.1.AND.L.LE.3) YA=TAN(TH)*TP*1.414
      YB=YA
      YC=YA
      Z11=(0.0,0.0)
      DO 10 I=1,NP
```

```
      ZI=-AL*0.5+HL*I
      FI=COS(PI*ZI/AL)
      Z1=(-ZL*0.5+(I-1)*HL)*TP
      Z2=Z1+HL*TP
      Z3=Z2+HL*TP
      DO 10 J=1,NP
      ZJ=-AL*0.5+HL*J
      FJ=COS(ZJ*PI/AL)
      ZA=(-ZL*0.5+(J-1)*HL)*TP+(L-1)*ZL*TP/NZ
      ZB=ZA+HL*TP
      ZC=ZB+HL*TP
      ZSS=ZIJ(TK,TK,TK,Y1,Y2,Y3,Z1,Z2,Z3,.0,.0,.0,YA,YB,YC,ZA,ZB,ZC)
     2*FI*FJ
      Z11=Z11+ZSS
   10 CONTINUE
      CL=K*DWL-1.414*V
      IF(V.LT.(K-1)*0.707*DWL) CL=1.414*V-(K-2)*DWL
      IF(K.NE.1) Z12=Z12+Z11*CC*DV*CL
      IF(K.EQ.1.AND.L.GT.3) Z12=Z12+Z11*CC*DV*CL
      IF(K.EQ.1.AND.L.LE.3) Z12=Z12+Z11*CC*DT*(DWL-1.414*TAN(TH))/
     2(COS(TH)*COS(TH))
   30 CONTINUE
      CK=2.
      IF(K.EQ.1.AND.L.LE.3) CK=4.
      Z(1,N1)=Z12*CK*0.707/3./(DWL*DWL)
   11 CONTINUE
      DO 40 M=1,NZS
      DO 40 N=1,NW
      I=N+(M-1)*NW
      MM(I)=M-1
      NN(I)=N-1
   40 CONTINUE
      DO 1 I=1,MODES
      DO 1 J=I,MODES
      N1=IABS(MM(J)-MM(I))
      N2=IABS(NN(J)-NN(I))
      DO 2 M=1,NZS
      DO 2 N=1,NW
      NI=N+(M-1)*NW
      IF(N1.EQ.M-1.AND.N2.EQ.N-1) GO TO 3
    2 CONTINUE
    3 CONTINUE
      Z(I,J)=Z(1,NI)
    1 CONTINUE
      DO 33 I=1,MODES
      DO 33 J=I,MODES
      II=I+MODES
      JJ=J+MODES
   33 Z(II,JJ)=Z(I,J)
C     THIS PART FOR CROSS-COUPLING
```

```
      NMAX=3
      DDW=DWL/(NMAX-1)
      CALL SIMWC(NMAX,C)
      DO 22 K=1,NW
      DO 22 L=1,NZS
      NI=K+(L-1)*NW
      DO 22 M=1,NW
      DO 22 N=1,NZS
      NJ=M+(N-1)*NW+MODES
      Z12=(0.0,0.0)
      DO 31 II=1,NMAX
      CI=C(II)
      X1=TK
      X2=TK
      X3=TK
      Y1=DDW*(II-1)*TP+(K-1)*DWL*TP
      Y2=Y1
      Y3=Y1
      DO 31 JJ=1,NMAX
      CJ=C(JJ)
      XA=0.0
      XB=0.0
      XC=0.0
      ZA=TP*(ZL-DDW*(JJ-1)-(M-1)*DWL)
      ZB=ZA
      ZC=ZA
      Z11=(0.0,0.0)
      DO 32 I=1,NP
      ZI=-0.5*AL+HL*I
      FI=COS(ZI*PI/AL)
      Z1=(I-1)*HL*TP+(L-1)*TP*ZL/NZ
      Z2=Z1+HL*TP
      Z3=Z2+HL*TP
      DO 32 J=1,NP
      ZJ=-0.5*AL+HL*J
      FJ=COS(ZJ*PI/AL)
      YA=(J-1)*HL*TP+(N-1)*TP*ZL/NZ
      YB=YA+HL*TP
      YC=YB+HL*TP
      ZSS=ZIJ(X1,X2,X3,Y1,Y2,Y3,Z1,Z2,Z3,XA,XB,XC,YA,YB,YC,ZA,ZB,ZC)
     2*FI*FJ
      Z11=ZSS+Z11
   32 CONTINUE
      Z12=Z12+Z11*CI*CJ
   31 CONTINUE
      Z(NI,NJ)=Z12*DDW*DDW/(DWL*DWL)/(3.**2)
       WRITE(6,99) NI,NJ,Z(NI,NJ)
   99  FORMAT(2I10,2F10.4)
   22 CONTINUE
```

```
C   THIS PART FIND THE VOLTAGES
       NMAX=15
       CALL SIMWC(NMAX,C)
       DDW=DWL/(NMAX-1)
      THI=90.
      PHI=0.0
 100   CONTINUE
      CTHI=COS(THI*PI/180.)
      STHI=SIN(THI*PI/180.)
      CPHI=COS(PHI*PI/180.)
      SPHI=SIN(PHI*PI/180.)
      DO 51 K=1,NW
      DO 51 L=1,NZS
      NI=K+(L-1)*NW
      NJ=NI+MODES
      VT(NI)=(0.0,0.0)
      VA=(0.0,0.0)
      VC=(0.0,0.0)
      DO 52 II=1,NMAX
      CI=C(II)
      X1=0.
      X2=0.
      Y1=DDW*(II-1)*TP+(K-1)*DWL*TP
      Y2=Y1
      ZA=TP*(ZL-DDW*(II-1)-(L-1)*DWL)
      ZB=ZA
      VD=(0.0,0.0)
      VB=(0.0,0.0)
      DO 53 I=1,NP
      ZI=-0.5*AL+HL*I
      FI=COS(ZI*PI/AL)
      Z1=(I-1)*HL*TP+(L-1)*TP*ZL/NZ
      Z2=Z1+HL*TP
      YA=(I-1)*HL*TP+(K-1)*TP*ZL/NZ
      YB=YA+HL*TP
      CALL ZFFD(X1,Y1,Z1,X2,Y2,Z2,HL*TP,CTHI,STHI,CPHI,SPHI,VTA,VPA)
      CALL ZFFD(X1,YA,ZA,X2,YB,ZB,HL*TP,CTHI,STHI,CPHI,SPHI,VTB,VPB)
      VB=VB+VTA*FI
      VD=VD+VTB*FI
   53 CONTINUE
      VC=VC+VD*DDW*CI
      VA=VA+VB*DDW*CI
   52 CONTINUE
      ET(NI)=VA/3./DWL
      ET(NJ)=VC/3./DWL
      VT(NI)=ET(NI)/CMPLX(0.0,-60.*PI)
      VT(NJ)=ET(NJ)/CMPLX(0.0,-60.*PI)
   51 CONTINUE
       IF (PHI .EQ. 0.0)  CALL CROUT(Z,VT,MODET,IDM,0,1,1)
       IF (PHI .EQ. 45.) CALL CROUT(Z,VT,MODET,IDM,0,1,2)
      EST=(0.0,0.0)
      DO 123 I=1,MODET
      EST=EST+ET(I)*VT(I)
```

```
123 CONTINUE
    ESTA=CABS(EST)
    ESTP=ATAN2(AIMAG(EST),REAL(EST))*180./PI
    SIG=4.*PI*ESTA*ESTA
    WRITE(6,124) ZL,SIG,ESTA,ESTP
124 FORMAT(5X,4F10.4)
     PHI=PHI+45.0
     IF (PHI .LE. 45.0) GO TO 100
     ZKL=ZKL+0.4
    END
    COMPLEX FUNCTION ZMN(DL,HL,SL)
    REAL L,LE,LL
    B=6.2831853
    D=DL
    L=HL
    LE=HL
    HC=SL
    BLE=B*LE
    H=ABS(HC)-L
    LL=LE
    HPL=H+LL
    HP2L=H+2.0*LL
    HP3L=H+3.0*LL
    HML=H-LL
    SBL=SIN(BLE)
    CBL=COS(BLE)
    SBH=SIN(B*H)
    CBH=COS(B*H)
    SBHML=SIN(B*HML)
    CBHML=COS(B*HML)
    SBHPL=SIN(B*HPL)
    CBHPL=COS(B*HPL)
    SBHP2L=SIN(B*HP2L)
    CBHP2L=COS(B*HP2L)
    SBHP3L=SIN(B*HP3L)
    CBHP3L=COS(B*HP3L)
    TEMP=SQRT(D*D+H*H)+H
    V1=B*D*D/TEMP
    U1=B*TEMP
    TEMP=SQRT(D*D+HML*HML)+HML
    U0=B*TEMP
    V0=B*D*D/TEMP
    TEMP=SQRT(D*D+HPL*HPL)+HPL
    U3=B*TEMP
    V3=B*D*D/TEMP
    TEMP=SQRT(D*D+HP2L*HP2L)+HP2L
    U2=B*D*D/TEMP
    V2=B*TEMP
    TEMP=SQRT(D*D+HP3L*HP3L)+HP3L
    U4=B*D*D/TEMP
    V4=B*TEMP
    CALL SICI (SIU0,CIU0,U0)
    CALL SICI (SIU1,CIU1,U1)
    CALL SICI (SIV2,CIV2,V2)
```

```
      CALL SICI (SIV4,CIV4,V4)
      CALL SICI (SIU3,CIU3,U3)
      IF (D.LE.0.0) GO TO 80
      CALL SICI (SIV1,CIV1,V1)
      CALL SICI (SIV0,CIV0,V0)
      CALL SICI (SIV3,CIV3,V3)
      CALL SICI (SIU2,CIU2,U2)
      CALL SICI (SIU4,CIU4,U4)
      R=15.0*(CBHML*(CIU0+CIV0-CIU1-CIV1)-SBHML*(-SIU0+SIV0+SIU1-SIV1)+C
     2BHPL*(2.*CIV3+2.*CIU3-CIU2-CIV2-CIU1-CIV1)+SBHPL*(-SIV3+SIU3+SIU2-
     3SIV2-SIU1+SIV1+SIU3-SIV3)+CBHP3L*(-CIU2-CIV2+CIU4+CIV4)+SBHP3L*(SI
     4U2-SIV2-SIU4+SIV4) +2.*CBL*CBH*(-CIV1-CIU1+CIV3+CIU3)+2.*CBL*SBH*(
     5SIV1-SIU1-SIV3+SIU3)+2.*CBL*CBHP2L*(CIV3+CIU3-CIU2-CIV2)+2.*CBL*SB
     6HP2L*(-SIV3+SIU3+SIU2-SIV2))
      X=15.0*(CBHML*(-SIU0-SIV0+SIU1+SIV1)-SBHML*(-CIU0+CIV0+CIU1-CIV1)+
     2CBHPL*(-2.*SIV3-2.*SIU3+SIU2+SIV2+SIU1+SIV1)+SBHPL*(-2.* CIV3+2.*C
     3IU3+CIU2-CIV2-CIU1+CIV1)+CBHP3L*(SIU2+SIV2-SIU4-SIV4)+SBHP3L*(CIU2
     4-CIV2-CIU4+CIV4)+2.*CBL*CBH*(SIV1+SIU1-SIV3-SIU3)+ 2.*CBL*SBH*(CIV
     51-CIU1-CIV3+CIU3)+2.*CBL*CBHP2L*(-SIV3-SIU3+SIU2+SIV2)+2.*CBL*SBHP
     62L*(-CIV3+CIU3+CIU2-CIV2))
      GO TO 90
   80 CONTINUE
      R=15.0*(CBHML*(CIU0-CIU1+ALOG(H /HML))+SBHML*(SIU0-SIU1)+SBHPL*
     2 (2.*SIU3 -SIV2-SIU1)+CBHPL*(2.*CIU3-CIV2-CIU1+ALOG(HP2L/HPL)+
     3ALOG(H /HPL))+CBHP3L*(CIV4-CIV2+ALOG(HP2L/HP3L))+SBHP3L*(SIV4-SIV2
     4 )+2.*CBL*CBH*(CIU3-CIU1+ALOG(H /HPL))+2.*CBL*SBH*(SIU3-SIU1) +
     5 2.*CBL*CBHP2L*(CIU3-CIV2+ALOG(HP2L/HPL))+2.*CBL*SBHP2L*(SIU3-
     6 SIV2))
      X=15.0*(CBHML*(SIU1-SIU0)+SBHML*(CIU0-CIU1+ALOG(HML/H ))+CBHPL*
     2 (SIV2+SIU1-2.*SIU3)+SBHPL *(2.*CIU3-CIV2-CIU1+ALOG(HPL/HP2L)+
     3ALOG(HPL/H ))+CBHP3L*(SIV2-SIV4)+SBHP3L*(CIV4-CIV2+ALOG(HP3L/HP2L)
     4)+2.*CBL*CBH*(SIU1-SIU3)+2.*CBL*SBH*(CIU3-CIU1+ALOG(HPL/H ))+2.*CB
     5L*CBHP2L*(SIV2-SIU3)+2.*CBL*SBHP2L*(CIU3-CIV2+ALOG(HPL/HP2L)))
   90 ZMN=CMPLX(R,X)/(SBL*SBL)
      RETURN
      END
      SUBROUTINE SICI(SI,CI,X)
      Z=ABS(X)
      IF(Z-4.0)1,1,4
    1 Y=(4.0-Z)*(4.0+Z)
      SI=-1.570797E0
      IF(Z)3,2,3
    2 CI=-1.0E38
      RETURN
    3 SI=X*((((((1.753141E-9*Y+1.568988E-7)*Y+1.374168E-5)*Y+6.939889E-4)
     2*Y+1.964882E-2)*Y+4.395509E-1+SI/X)
      CI=((5.772156E-1+ALOG(Z))/Z-Z*(((((1.386985E-10*Y+1.584996E-8)*Y
     2+1.725752E-6)*Y+1.185999E-4)*Y+4.990920E-3)*Y+1.315308E-1))*Z
      RETURN
```

```
    4 SI=SIN(Z)
      Y=COS(Z)
      Z=4.0/Z
      U=((((((((4.048069E-3*Z-2.279143E-2)*Z+5.515070E-2)*Z-7.261642E-2)
     2*Z+4.987716E-2)*Z-3.332519E-3)*Z-2.314617E-2)*Z-1.134958E-5)*Z
     3+6.250011E-2)*Z+2.583989E-10
      V=(((((((((-5.108699E-3*Z+2.819179E-2)*Z-6.537283E-2)*Z
     2+7.902034E-2)*Z-4.400416E-2)*Z-7.945556E-3)*Z+2.601293E-2)*Z
     3-3.764000E-4)*Z-3.122418E-2)*Z-6.646441E-7)*Z+2.500000E-1
      CI=Z*(SI*V-Y*U)
      SI=-Z*(SI*U+Y*V)
      IF(X)5,6,6
    5 SI=-3.141593E0-SI
    6 RETURN
      END
      SUBROUTINE CROUT(C,S,N,IDM,ISYM,IWR,I12)
      COMPLEX C(IDM,IDM),S(IDM)
      COMPLEX F,P,SS,T
  2   FORMAT(1X,I15,1F10.3,1F15.7,1F10.0)
  5   FORMAT(1H0)
      IF(I12.NE.1)GO TO 22
      IF(N.EQ.1)S(1)=S(1)/C(1,1)
      IF(N.EQ.1)GO TO 100
      IF(ISYM.NE.0)GO TO 8
      DO 6 I=1,N
      DO 6 J=I,N
      C(J,I)=C(I,J)
  6   CONTINUE
  8   CONTINUE
      F=C(1,1)
      DO 10 L=2,N
  10  C(1,L)=C(1,L)/F
      DO 20 L=2,N
      LLL=L-1
      DO 20 I=L,N
      F=C(I,L)
      DO 11 K=1,LLL
  11  F=F-C(I,K)*C(K,L)
      C(I,L)=F
      IF(L.EQ.I)GO TO 20
      P=C(L,L)
      IF(ISYM.EQ.0)GO TO 15
      F=C(L,I)
      DO 12 K=1,LLL
  12  F=F-C(L,K)*C(K,I)
      C(L,I)=F/P
      GO TO 20
  15  F=C(I,L)
      C(L,I)=F/P
  20  CONTINUE
  22  CONTINUE
```

```
      DO 30 L=1,N
      P=C(L,L)
      T=S(L)
      IF(L.EQ.1)GO TO 30
      LLL=L-1
      DO 25 K=1,LLL
25    T=T-C(L,K)*S(K)
30    S(L)=T/P
      DO 38 L=2,N
      I=N-L+1
      II=I+1
      T=S(I)
      DO 35 K=II,N
35    T=T-C(I,K)*S(K)
38    S(I)=T
      IF(IWR.LE.0) GO TO 100
      CNOR=.0
      DO 40 I=1,N
      SA=CABS(S(I))
      IF(SA.GT.CNOR)CNOR=SA
40    CONTINUE
      IF(CNOR.LE.0.)CNOR=1.
      DO 44 I=1,N
      SS=S(I)
      SA=CABS(SS)
      SNOR=SA/CNOR
      PH=.0
      IF(SA.GT.0.)PH=57.29578*ATAN2(AIMAG(SS),REAL(SS))
      WRITE(6,2)I,SNOR,SA,PH
44    CONTINUE
      WRITE(6,5)
100   CONTINUE
      RETURN
      END
      COMPLEX FUNCTION ZIJ(X1,X2,X3,Y1,Y2,Y3,Z1,Z2,Z3,XA,XB,XC,YA,YB,YC,
     2ZA,ZB,ZC)
      COMPLEX P11,P12,P21,P22
      COMPLEX Q11,Q12,Q21,Q22
      COMPLEX S11,S12,S21,S22
      COMPLEX R11,R12,R21,R22
      AK=0.005*2.0*3.141592
      INT=0
      D12=SQRT((X1-X2)*(X1-X2)+(Y1-Y2)*(Y1-Y2)+(Z1-Z2)*(Z1-Z2))
      D23=SQRT((X2-X3)*(X2-X3)+(Y2-Y3)*(Y2-Y3)+(Z2-Z3)*(Z2-Z3))
      DAB=SQRT((XA-XB)*(XA-XB)+(YA-YB)*(YA-YB)+(ZA-ZB)*(ZA-ZB))
      DCB=SQRT((XC-XB)*(XC-XB)+(YC-YB)*(YC-YB)+(ZC-ZB)*(ZC-ZB))
      CD12=COS(D12)
      SD12=SIN(D12)
      CD23=COS(D23)
      SD23=SIN(D23)
      CDAB=COS(DAB)
      SDAB=SIN(DAB)
```

```
      CDCB=COS(DCB)
      SDCB=SIN(DCB)
      CALL ZGS(X1,Y1,Z1,X2,Y2,Z2,XA,YA,ZA,XB,YB,ZB,AK,D12,CD12,SD12,
     2DAB,SDAB,INT,P11,P12,P21,P22)
      CALL ZGS(X1,Y1,Z1,X2,Y2,Z2,XB,YB,ZB,XC,YC,ZC,AK,D12,CD12,SD12,
     2DCB,SDCB,INT,Q11,Q12,Q21,Q22)
      CALL ZGS(X2,Y2,Z2,X3,Y3,Z3,XA,YA,ZA,XB,YB,ZB,AK,D23,CD23,SD23,
     2DAB,SDAB,INT,R11,R12,R21,R22)
      CALL ZGS(X2,Y2,Z2,X3,Y3,Z3,XB,YB,ZB,XC,YC,ZC,AK,D23,CD23,SD23,
     2DCB,SDCB,INT,S11,S12,S21,S22)
      ZIJ=P22+Q21+R12+S11
      RETURN
      END
      SUBROUTINE SIMWC(NMAX,C)
      DIMENSION C(31)
      DO 1 N=1,NMAX
      XNN=FLOAT(N)
      NN=N/2
      TT=XNN/2.
      DIF=TT-FLOAT(NN)
      NC=2
      IF(DIF.EQ.0.) NC=4
      IF(N.EQ.1.OR.N.EQ.NMAX) NC=1
      C(N)=NC
    1 CONTINUE
      RETURN
      END
      SUBROUTINE ZFFD(XA,YA,ZA,XB,YB,ZB,D,CTH,STH,CPH,SPH,ET,EP)
      COMPLEX ET,EP,ES,EJA,EJB
      XAB=XB-XA
      YAB=YB-YA
      ZAB=ZB-ZA
      CA=XAB/D
      CB=YAB/D
      CG=ZAB/D
      G=(CA*CPH+CB*SPH)*STH+CG*CTH
      GK=1.-G*G
      ET=(0.0,0.0)
      EP=(0.0,0.0)
      IF(GK.LT.0.001) GO TO 200
      B=XB*STH*CPH+YB*STH*SPH+ZB*CTH
      EJB=CMPLX(COS(B),SIN(B))
      SKD=SIN(D)
      CKD=COS(D)
      CGD=COS(G*D)
      ES=60.0*(.0,1.)*EJB*(CKD-CGD)/SKD/GK
      T=(CA*CPH+CB*SPH)*CTH-CG*STH
      P=-CA*SPH+CB*CPH
      ET=ES*T
      EP=ES*P
  200 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE ZGS(XA,YA,ZA,XB,YB,ZB,X1,Y1,Z1,X2,Y2,Z2,AK,
     2DS,CDS,SDS,DT,SDT,INT,P11,P12,P21,P22)
      COMPLEX CST,EJ1,EJ2,EJA,EJB,ER1,ER2,ET1,ET2,P11,P12,P21,P22,GAM
      COMPLEX SGDS,SGDT
      DATA ETA,GAM,PI/376.727,(.0,1.),3.14159/
      CA=(X2-X1)/DT
      CB=(Y2-Y1)/DT
      CG=(Z2-Z1)/DT
      CAS=(XB-XA)/DS
      CBS=(YB-YA)/DS
      CGS=(ZB-ZA)/DS
      CC=CA*CAS+CB*CBS+CG*CGS
      IF(ABS(CC).GT.0.997)GO TO 200
      SZ=(X1-XA)*CAS+(Y1-YA)*CBS+(Z1-ZA)*CGS
      IF(INT.EQ.0)GO TO 300
      CGDS=CDS
      SGDS=CMPLX(.0,SDS)
      SGDT=CMPLX(.0,SDT)
      INS=2*(INT/2)
      IF(INS.LT.2)INS=2
      IP=INS+1
      DELT=DT/INS
      T=.0
      DSZ=CC*DELT
      P11=(.0,.0)
      P12=(.0,.0)
      P21=(.0,.0)
      P22=(.0,.0)
      AKS=AK*AK
      SGN=-1.
      DO 100 IN=1,IP
      ZZ1=SZ
      ZZ2=SZ-DS
      XXZ=X1+T*CA-XA-SZ*CAS
      YYZ=Y1+T*CB-YA-SZ*CBS
      ZZZ=Z1+T*CG-ZA-SZ*CGS
      RS=XXZ**2+YYZ**2+ZZZ**2
      R1=SQRT(RS+ZZ1**2)
      EJA=CMPLX(COS(R1),-SIN(R1))
      EJ1=EJA/R1
      R2=SQRT(RS+ZZ2**2)
      EJB=CMPLX(COS(R2),-SIN(R2))
      EJ2=EJB/R2
      ER1=EJA*SGDS+ZZ1*EJ1*CGDS-ZZ2*EJ2
      ER2=-EJB*SGDS+ZZ2*EJ2*CGDS-ZZ1*EJ1
      FAC=.0
      IF(RS.GT.AKS)FAC=(CA*XXZ+CB*YYZ+CG*ZZZ)/RS
      ET1=CC*(EJ2-EJ1*CGDS)+FAC*ER1
      ET2=CC*(EJ1-EJ2*CGDS)+FAC*ER2
      C=3.+SGN
      IF(IN.EQ.1 .OR. IN.EQ.IP)C=1.
```

```
      C1=C*SIN(DT-T)
      C2=C*SIN(T)
      P11=P11+ET1*C1
      P12=P12+ET1*C2
      P21=P21+ET2*C1
      P22=P22+ET2*C2
      T=T+DELT
      SZ=SZ+DSZ
100   SGN=-SGN
      CST=-(.0,1.)*ETA*DELT/(12.*PI*SGDS*SGDT)
      P11=CST*P11
      P12=CST*P12
      P21=CST*P21
      P22=CST*P22
      RETURN
200   SZ1=(X1-XA)*CAS+(Y1-YA)*CBS+(Z1-ZA)*CGS
      RH1=SQRT((X1-XA-SZ1*CAS)**2+(Y1-YA-SZ1*CBS)**2+(Z1-ZA-SZ1*CGS)**2)
      SZ2=SZ1+DT*CC
      RH2=SQRT((X2-XA-SZ2*CAS)**2+(Y2-YA-SZ2*CBS)**2+(Z2-ZA-SZ2*CGS)**2)
      DDK=(RH1+RH2)/2.
      IF(DDK.LT.AK)DDK=AK
      CALL ZGMM(.0,DS,SZ1,SZ2,DDK,CDS,SDS,SDT,1.,P11,P12,P21,P22)
      RETURN
300   SS=SQRT(1.-CC*CC)
      CAD=(CGS*CB-CBS*CG)/SS
      CBD=(CAS*CG-CGS*CA)/SS
      CGD=(CBS*CA-CAS*CB)/SS
      DK=(X1-XA)*CAD+(Y1-YA)*CBD+(Z1-ZA)*CGD
      DK=ABS(DK)
      IF(DK.LT.AK)DK=AK
      XZ=XA+SZ*CAS
      YZ=YA+SZ*CBS
      ZZ=ZA+SZ*CGS
      XP1=X1-DK*CAD
      YP1=Y1-DK*CBD
      ZP1=Z1-DK*CGD
      CAP=CBS*CGD-CGS*CBD
      CBP=CGS*CAD-CAS*CGD
      CGP=CAS*CBD-CBS*CAD
      P1=CAP*(XP1-XZ)+CBP*(YP1-YZ)+CGP*(ZP1-ZZ)
      T1=P1/SS
      S1=T1*CC-SZ
      CALL ZGMM(S1,S1+DS,T1,T1+DT,DK,CDS,SDS,SDT,CC,P11,P12,P21,P22)
      RETURN
      END
      SUBROUTINE EXPJ(V1,V2,W12)
      COMPLEX EC,E15,S,T,UC,VC,V1,V2,W12,Z
      DIMENSION V(21),W(21),D(16),E(16)
      DATA V/   0.22284667E 00,
     20.11889321E 01,0.29927363E 01,0.57751436E 01,0.98374674E 01,
     20.15982874E 02,0.93307812E-01,0.49269174E 00,0.12155954E 01,
     20.22699495E 01,0.36676227E 01,0.54253366E 01,0.75659162E 01,
     20.10120228E 02,0.13130282E 02,0.16654408E 02,0.20776479E 02,
     20.25623894E 02,0.31407519E 02,0.38530683E 02,0.48026086E 02/
```

```
      DATA W/   0.45896460E 00,
     20.41700083E 00,0.11337338E 00,0.10399197E-01,0.26101720E-03,
     20.89854791E-06,0.21823487E 00,0.34221017E 00,0.26302758E 00,
     20.12642582E 00,0.40206865E-01,0.85638778E-02,0.12124361E-02,
     20.11167440E-03,0.64599267E-05,0.22263169E-06,0.42274304E-08,
     20.39218973E-10,0.14565152E-12,0.14830270E-15,0.16005949E-19/
      DATA D/   0.22495842E 02,
     2 0.74411568E 02,-0.41431576E 03,-0.78754339E 02, 0.11254744E 02,
     2 0.16021761E 03,-0.23862195E 03,-0.50094687E 03,-0.68487854E 02,
     2 0.12254778E 02,-0.10161976E 02,-0.47219591E 01, 0.79729681E 01,
     2-0.21069574E 02, 0.22046490E 01, 0.89728244E 01/
      DATA E/   0.21103107E 02,
     2-0.37959787E 03,-0.97489220E 02, 0.12900672E 03, 0.17949226E 02,
     2-0.12910931E 03,-0.55705574E 03, 0.13524801E 02, 0.14696721E 03,
     2 0.17949528E 02,-0.32981014E 00, 0.31028836E 02, 0.81657657E 01,
     2 0.22236961E 02, 0.39124892E 02, 0.81636799E 01/
      Z=VI
      DO 100 JIM=1,2
      X=REAL(Z)
      Y=AIMAG(Z)
      E15=(.0,.0)
      AB=CABS(Z)
      IF(AB.EQ.0.)GO TO 90
      IF(X.GE.0. .AND. AB.GT.10.)GO TO 80
      YA=ABS(Y)
      IF(X.LE.0. .AND. YA.GT.10.)GO TO 80
      IF(YA-X.GE.17.5.OR.YA.GE.6.5.OR.X+YA.GE.5.5.OR.X.GE.3.)GO TO 20
      IF(X.LE.-9.)GO TO 40
      IF(YA-X.GE.2.5)GO TO 50
      IF(X+YA.GE.1.5)GO TO 30
   10 N=6.+3.*AB
      E15=1./(N-1.)-Z/N**2
   15 N=N-1
      E15=1./(N-1.)-Z*E15/N
      IF(N.GE.3)GO TO 15
      E15=Z*E15-CMPLX(.577216+ALOG(AB),ATAN2(Y,X))
      GO TO 90
   20 J1=1
      J2=6
      GO TO 31
   30 J1=7
      J2=21
   31 S=(.0,.0)
      YS=Y*Y
      DO 32 I=J1,J2
      XI=V(I)+X
      CF=W(I)/(XI*XI+YS)
   32 S=S+CMPLX(XI*CF,-YA*CF)
      GO TO 54
   40 T3=X*X-Y*Y
      T4=2.*X*YA
      T5=X*T3-YA*T4
      T6=X*T4+YA*T3
```

```
      UC=CMPLX(D(11)+D(12)*X+D(13)*T3+T5-E(12)*YA-E(13)*T4,
     2        F(11)+F(12)*X+F(13)*T3+T6+D(12)*YA+D(13)*T4)
      VC=CMPLX(D(14)+D(15)*X+D(16)*T3+T5-E(15)*YA-E(16)*T4,
     2        F(14)+F(15)*X+F(16)*T3+T6+D(15)*YA+D(16)*T4)
      GO TO 52
   50 T3=X*X-Y*Y
      T4=2.*X*YA
      T5=X*T3-YA*T4
      T6=X*T4+YA*T3
      T7=X*T5-YA*T6
      T8=X*T6+YA*T5
      T9=X*T7-YA*T8
      T10=X*T8+YA*T7
      UC=CMPLX(D(1)+D(2)*X+D(3)*T3+D(4)*T5+D(5)*T7+T9-(E(2)*YA+E(3)*T4
     2+E(4)*T6+E(5)*T8),E(1)+E(2)*X+E(3)*T3+E(4)*T5+E(5)*T7+T10+
     3(D(2)*YA+D(3)*T4+D(4)*T6+D(5)*T8))
      VC=CMPLX(D(6)+D(7)*X+D(8)*T3+D(9)*T5+D(10)*T7+T9-(E(7)*YA+E(8)*T4
     2+E(9)*T6+E(10)*T8),E(6)+E(7)*X+E(8)*T3+E(9)*T5+E(10)*T7+T10+
     3(D(7)*YA+D(8)*T4+D(9)*T6+D(10)*T8))
   52 EC=UC/VC
      S=EC/CMPLX(X,YA)
   54 EX=EXP(-X)
      T=EX*CMPLX(COS(YA),-SIN(YA))
      E15=S*T
   56 IF(Y.LT.0.)E15=CONJG(E15)
      GO TO 90
   80 E15=.409319/(Z+.193044)+.421831/(Z+1.02666)+.147126/(Z+2.56788)+
     2.206335E-1/(Z+4.90035)+.107401E-2/(Z+8.18215)+.158654E-4/(Z+
     312.7342)+.317031E-7/(Z+19.3957)
      E15=E15*CEXP(-Z)
   90 IF(JIM.EQ.1)W12=E15
  100 Z=V2
      Z=V2/V1
      TH=ATAN2(AIMAG(Z),REAL(Z))-ATAN2(AIMAG(V2),REAL(V2))
     2+ATAN2(AIMAG(V1),REAL(V1))
      AB=ABS(TH)
      IF(AB.LT.1.)TH=.0
      IF(TH.GT.1.)TH=6.2831853
      IF(TH.LT.-1.)TH=-6.2831853
      W12=W12-E15+CMPLX(.0,TH)
      RETURN
      END
      SUBROUTINE ZGMM(S1,S2,T1,T2,D,CGDS,SGD1,SGD2,CPSI,P11,P12,P21,P22)
      COMPLEX E(2,2),F(2,2),GAM,P11,P12,P21,P22
      COMPLEX EB,EC,EK,EL,EKL,EGZI,ES1,ES2,ET1,ET2,EXPA,EXPB
      COMPLEX EGZ(2,2),GM(2),GP(2)
      COMPLEX EXA(2),EXB(2)
      DATA ETA,GAM,PI/376.727,(.0,1.),3.14159/
      DSQ=D*D
      SGDS=SGD1
      IF(S2.LT.S1)SGDS=-SGD1
```

```
      SGDT=SGD2
      IF(T2.LT.T1)SGDT=-SGD2
      IF(ABS(CPSI).GT.0.997)GO TO 110
      ES1=CEXP(GAM*S1)
      ES2=CEXP(GAM*S2)
      ET1=CEXP(GAM*T1)
      ET2=CEXP(GAM*T2)
      C=D/SQRT(1.-CPSI*CPSI)
      B=C*CPSI
      EB=CEXP(GAM*CMPLX(.0,B))
      EC=CEXP(GAM*CMPLX(.0,C))
      DO 10 K=1,2
      DO 10 L=1,2
   10 E(K,L)=(.0,.0)
      EK=EB
      DO 50 K=1,2
      FK=(-1)**K
      EL=EC
      DO 40 L=1,2
      FL=(-1)**L
      EKL=EK*EL
      XX=FK*B+FL*C
      SI=S1
      DO 30 I=1,2
      R1=SQRT(DSQ+SI*SI+T1*T1-2.*SI*T1*CPSI)
      R2=SQRT(DSQ+SI*SI+T2*T2-2.*SI*T2*CPSI)
      CALL EXPJ(GAM*CMPLX(R1+FK*SI+FL*T1,-XX),
     2          GAM*CMPLX(R2+FK*SI+FL*T2,-XX),EXA(I))
      CALL EXPJ(GAM*CMPLX(R1+FK*SI+FL*T1,XX),
     2          GAM*CMPLX(R2+FK*SI+FL*T2,XX),EXB(I))
      IF(K.EQ.2 .OR. L.EQ.2)GO TO 30
      ZC=SI*CPSI
      EGZI=CEXP(GAM*ZC)
      CALL EXPJ(GAM*(R1+ZC-T1),GAM*(R2+ZC-T2),EXPB)
      CALL EXPJ(GAM*(R1-ZC+T1),GAM*(R2-ZC+T2),EXPA)
      F(I,1)=2.*SGDS*(.0,1.)*EXPA/EGZI
      F(I,2)=2.*SGDS*(.0,1.)*EXPB*EGZI
   30 SI=S2
      E(K,L)=E(K,L)+(EXA(2)-EXA(1))*EKL+(EXB(2)-EXB(1))/EKL
   40 EL=1./EC
   50 EK=1./EB
      CST=-ETA/(16.*PI*SGDS*SGDT)
      P11=CST*(( F(1,1)+E(2,2)*ES2-E(1,2)/ES2)*ET2
     A        +(-F(1,2)-E(2,1)*ES2+E(1,1)/ES2)/ET2)
      P12=CST*((-F(1,1)-E(2,2)*ES2+E(1,2)/ES2)*ET1
     B        +( F(1,2)+E(2,1)*ES2-E(1,1)/ES2)/ET1)
      P21=CST*((-F(2,1)-E(2,2)*ES1+E(1,2)/ES1)*ET2
     C        +( F(2,2)+E(2,1)*ES1-E(1,1)/ES1)/ET2)
      P22=CST*(( F(2,1)+E(2,2)*ES1-E(1,2)/ES1)*ET1
     D        +(-F(2,2)-E(2,1)*ES1+E(1,1)/ES1)/ET1)
      RETURN
```

```
110 IF(CPSI.LT.0.)GO TO 120
    TA=T1
    TB=T2
    GO TO 130
120 TA=-T1
    TB=-T2
    SGDT=-SGDT
130 SI=S1
    DO 150 I=1,2
    TJ=TA
    DO 140 J=1,2
    ZIJ=TJ-SI
    R=SQRT(DSQ+ZIJ*ZIJ)
    W=R+ZIJ
    IF(ZIJ.LT.0.)W=DSQ/(R-ZIJ)
    V=R-ZIJ
    IF(ZIJ.GT.0.)V=DSQ/(R+ZIJ)
    IF(J.EQ.1)V1=V
    IF(J.EQ.1)W1=W
    EGZ(I,J)=CEXP(GAM*ZIJ)
140 TJ=TB
    CALL EXPJ(GAM*V1,GAM*V,GP(I))
    CALL EXPJ(GAM*W1,GAM*W,GM(I))
150 SI=S2
    CST=ETA/(8.*PI*SGDS*SGDT)
    P11=CST*(GM(2)*EGZ(2,2)+GP(2)/EGZ(2,2)
   2-CGDS*(GM(1)*EGZ(1,2)+GP(1)/EGZ(1,2)))
    P12=CST*(-GM(2)*EGZ(2,1)-GP(2)/EGZ(2,1)
   2+CGDS*(GM(1)*EGZ(1,1)+GP(1)/EGZ(1,1)))
    P21=CST*(GM(1)*EGZ(1,2)+GP(1)/EGZ(1,2)
   2-CGDS*(GM(2)*EGZ(2,2)+GP(2)/EGZ(2,2)))
    P22=CST*(-GM(1)*EGZ(1,1)-GP(1)/EGZ(1,1)
   2+CGDS*(GM(2)*EGZ(2,1)+GP(2)/EGZ(2,1)))
    RETURN
    END
```